

# On convergence of dynamics of hopping particles to a birth-and-death process in continuum

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## Abstract

We show that some classes of birth-and-death processes in continuum (Glauber dynamics) may be derived as a scaling limit of a dynamics of interacting hopping particles (Kawasaki dynamics)

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## 1 Preliminaries

This letter deals with two classes of stochastic dynamics of infinite particle systems in continuum. Let  $\Gamma$  denote the space of all locally finite subsets of  $\mathbb{R}^d$ ,  $d \in \mathbb{N}$ . This space is called the configuration space. Elements of  $\Gamma$  are called configurations, and each point of a configuration represents position of a particle. We endow  $\Gamma$  with the vague topology, i.e., the weakest topology in  $\Gamma$  with respect to which every mapping of the form  $\Gamma \ni \gamma \mapsto \langle f, \gamma \rangle := \sum_{x \in \gamma} f(x)$ , with  $f \in C_0(\mathbb{R}^d)$ , is continuous. Here  $C_0(\mathbb{R}^d)$  is the space of all real-valued functions on  $\mathbb{R}^d$  with compact support. We denote by  $\mathcal{B}(\Gamma)$  the Borel  $\sigma$ -algebra in  $\Gamma$ .

A dynamics of hopping particles (Kawasaki dynamics) is a Markov process on  $\Gamma$  whose generator is given (on an appropriate set of functions on  $\Gamma$ ) by

$$(L_K F)(\gamma) = \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy c(x, y, \gamma \setminus x) (F(\gamma \setminus x \cup y) - F(\gamma)).$$

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Here and below, for simplicity of notations, we just write  $x, y$  instead of  $\{x\}, \{y\}$ . The function  $c(x, y, \gamma \setminus x)$  describes the rate at which a particle  $x$  of configuration  $\gamma$  jumps to  $y$ , taking into account the rest of configuration,  $\gamma \setminus x$ .

A birth-and-death process in continuum (Glauber dynamics) is a Markov process on  $\Gamma$  with generator

$$(L_G F)(\gamma) = \sum_{x \in \gamma} d(x, \gamma \setminus x)(F(\gamma \setminus x) - F(\gamma)) + \int_{\mathbb{R}^d} dy b(y, \gamma)(F(\gamma \cup y) - F(\gamma)).$$

Here  $d(x, \gamma \setminus x)$  describes the rate at which a particle  $x$  of configuration  $\gamma$  dies, whereas  $b(y, \gamma)$  describes the rate at which, given configuration  $\gamma$ , a new particle is born at  $y$ . For some constructions and discussions of Glauber and Kawasaki dynamics in continuum, see [1, 3, 4, 11, 12, 13, 14, 16] and the references therein.

The aim of this letter is to show that, in many cases, a birth-and-death process may be interpreted as a limiting dynamics of hopping particles. We will restrict our attention to the case where the rate  $c$  of the Kawasaki dynamics is given by

$$c(x, y, \gamma \setminus x) = a(x - y) \exp[E^{\phi^-}(x, \gamma \setminus x) - E^{\phi^+}(y, \gamma \setminus x)].$$

Here  $a$  and  $\phi^\pm$  are even functions on  $\mathbb{R}^d$  (e.g.  $a(-x) = a(x)$ ),  $a$  is bounded,  $a \geq 0$ ,  $\int_{\mathbb{R}^d} a(x) dx = 1$ , and for  $x \in \mathbb{R}^d$  and  $\gamma \in \Gamma$ ,

$$E^{\phi^\pm}(x, \gamma) := \sum_{y \in \gamma} \phi^\pm(x - y),$$

provided the sum converges absolutely. Thus,  $c(x, y, \gamma \setminus x)$  is a product of three terms: the term  $e^{E^{\phi^-}(x, \gamma \setminus x)}$  describes the rate at which a particle  $x \in \gamma$  jumps, the term  $e^{-E^{\phi^+}(y, \gamma \setminus x)}$  describes the rate at which this particle lands at  $y$ , and finally the term  $a(x - y)$  gives the distribution of an individual jump.

We now produce the following scaling of this dynamics. For each  $\varepsilon > 0$ , we define  $a_\varepsilon(x) := \varepsilon^d a(\varepsilon x)$ . We clearly have that  $\int_{\mathbb{R}^d} a_\varepsilon(x) dx = 1$ . Let  $c_\varepsilon$  denote the  $c$  coefficient in which function  $a$  is replaced by  $a_\varepsilon$ , and let  $L_\varepsilon$  denote the corresponding  $L_K$  generator. Letting  $\varepsilon \rightarrow 0$ , we may suggest that only jumps of infinite length will survive, i.e., jumps from a point to ‘infinity’, and jumps from ‘infinity’ to a point. Thus, we expect to arrive at a birth-and-death process. To make our suggestion more explicit, we proceed as follows.

## 2 Convergence of the generator of the scaled evolution of correlation functions

For simplicity, we assume, in this section, that the functions  $\phi^\pm$  are from  $C_0(\mathbb{R}^d)$ . Then  $E^{\phi^\pm}(x, \gamma)$  are well defined for each  $x \in \mathbb{R}^d$  and  $\gamma \in \Gamma$ .

Let us briefly recall some basic facts of harmonic analysis on the configuration space, see [8, 10] for further detail. Let  $\Gamma_0$  denote the space of all finite configurations in  $\mathbb{R}^d$ , i.e.,  $\Gamma_0 = \bigcup_{n=0}^{\infty} \Gamma^{(n)}$ , where  $\Gamma^{(n)}$  is the space of all  $n$ -point configurations in  $\mathbb{R}^d$ . Clearly,  $\Gamma_0 \subset \Gamma$ , and we define  $\mathcal{B}(\Gamma_0)$  and  $\mathcal{B}(\Gamma^{(n)})$  as the trace  $\sigma$ -algebra of  $\Gamma$  on  $\Gamma_0$  and  $\Gamma^{(n)}$ , respectively. For a function  $G : \Gamma_0 \rightarrow \mathbb{R}$ , we define a function  $(KG)(\gamma) := \sum_{\eta \in \gamma} G(\eta)$ ,  $\gamma \in \Gamma$ , provided the summation makes sense. Here  $\eta \in \gamma$  means that  $\eta$  is a finite subset of  $\gamma$ .

Let  $\mu$  be a probability measure on  $(\Gamma, \mathcal{B}(\Gamma))$ . Then there exists a unique measure  $\rho$  on  $(\Gamma_0, \mathcal{B}(\Gamma_0))$  satisfying

$$\int_{\Gamma} (KG)(\gamma) \mu(d\gamma) = \int_{\Gamma_0} G(\eta) \rho(d\eta)$$

for each measurable function  $G : \Gamma_0 \rightarrow [0, \infty)$ . The measure  $\rho$  is called the correlation measure of  $\mu$ . Further, denote by  $\lambda$  the Lebesgue–Poisson measure on  $\Gamma_0$ , i.e.,

$$\lambda = \delta_{\emptyset} + \sum_{n=1}^{\infty} \frac{1}{n!} dx_1 \cdots dx_n.$$

Here  $\delta_{\emptyset}$  is the Dirac measure with mass at  $\emptyset$ , and  $dx_1 \cdots dx_n$  is the Lebesgue measure on  $\Gamma^{(n)}$ , which is naturally defined on this space. Assume that the correlation measure  $\rho$  of  $\mu$  is absolutely continuous with respect to  $\lambda$ . Then  $k := \frac{d\rho}{d\lambda}$  is called the correlation functional of  $\mu$ . For a given correlation functional  $k$ , the corresponding Ursell functional  $u : \Gamma_0 \rightarrow \mathbb{R}$  is defined through the formula  $k(\eta) = \sum_{\pi \in \mathcal{P}(\eta)} u_{\pi}(\eta)$ , where  $\mathcal{P}(\eta)$  denotes the set of all partitions of  $\eta$ , and given a partition  $\pi = \{\eta_1, \dots, \eta_k\}$  of  $\eta$ ,  $u_{\pi}(\eta) := u(\eta_1) \cdots u(\eta_k)$ . Recall also that a function  $G : \Gamma_0 \rightarrow \mathbb{R}$  is called translation invariant if, for each  $x \in \mathbb{R}^d$ ,  $G(\eta_x) = G(\eta)$  for all  $\eta \in \Gamma_0$ , where  $\eta_x$  denotes the configuration  $\eta$  shifted by vector  $x$ , i.e.,  $\eta_x := \{y + x \mid y \in \eta\}$ . Clearly, the correlation functional  $k$  is translation invariant if and only if the corresponding Ursell functional  $u$  is translation invariant.

If  $k$  is the correlational functional of a probability measure  $\mu$  on  $\Gamma$ , we denote

$$k^{(n)}(x_1, \dots, x_n) := k(\{x_1, \dots, x_n\}), \quad n \in \mathbb{N},$$

and analogously we define  $u^{(n)}$ . The  $(k^{(n)})_{n=1}^{\infty}$  and  $(u^{(n)})_{n=1}^{\infty}$  are called the correlation and Ursell functions of  $\mu$ , respectively. Note that, if  $k$  is translation invariant, then  $k^{(1)} = u^{(1)}$  is a constant.

For a function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$ , we define  $e_{\lambda}(f, \eta) := \prod_{x \in \eta} f(x)$ ,  $\eta \in \Gamma_0$ , where  $\prod_{x \in \emptyset} f(x) := 1$ . Further, let  $\varphi : \mathbb{R}^d \rightarrow \mathbb{R}$ . Then

$$(Ke_{\lambda}(e^{\varphi} - 1, \cdot))(\gamma) = e^{\langle \varphi, \gamma \rangle},$$

so that

$$\int_{\Gamma} e^{\langle \varphi, \gamma \rangle} \mu(d\gamma) = \int_{\Gamma_0} e_{\lambda}(e^{\varphi} - 1, \eta) k(\eta) \lambda(d\eta), \quad (1)$$

under some proper conditions on  $\varphi$  and  $k$ , see e.g. [10].

Assume that  $L$  is a Markov generator on  $\Gamma$ . Denote  $\hat{L} := K^{-1}LK$ , i.e.,  $\hat{L}$  is the operator acting on functions on  $\Gamma_0$  which satisfies  $K\hat{L}G = LKG$ . Denote by  $\hat{L}^*$  the dual operator of  $\hat{L}$  with respect to the Lebesgue–Poisson measure  $\lambda$ :

$$\int_{\Gamma_0} (\hat{L}G)(\eta)k(\eta)\lambda(d\eta) = \int_{\Gamma_0} G(\eta)(\hat{L}^*k)(\eta)\lambda(d\eta).$$

Assume now that a Markov process on  $\Gamma$  with generator  $L$  has initial distribution  $\mu_0$ . Denote by  $\mu_t$  the distribution of this process at time  $t > 0$ . Assume that, for each  $t \geq 0$ ,  $\mu_t$  has correlation functional  $k_t$ . Then, at least at an informal level, one sees that the evolution of  $k_t$  is described by the equation  $\partial k_t / \partial t = \hat{L}^*k_t$ , so that  $\hat{L}^*$  is the generator of evolution of correlation functionals.

In the case where  $L = L_\varepsilon$ , we proceed as follows. First we write  $L_\varepsilon = L_\varepsilon^- + L_\varepsilon^+$ , where

$$\begin{aligned} (L_\varepsilon^- F)(\gamma) &= \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy a_\varepsilon(x-y) r(x, y, \gamma \setminus x) (F(\gamma \setminus x) - F(\gamma)), \\ (L_\varepsilon^+ F)(\gamma) &= \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy a_\varepsilon(x-y) r(x, y, \gamma \setminus x) (F(\gamma \setminus x \cup y) - F(\gamma \setminus x)). \end{aligned}$$

Here,  $r(x, y, \gamma \setminus x) := \exp[E^{\phi^-}(x, \gamma \setminus x) - E^{\phi^+}(y, \gamma \setminus x)]$ . We also set

$$\begin{aligned} (L_0^- F)(\gamma) &= \sum_{x \in \gamma} \exp[E^{\phi^-}(x, \gamma \setminus x)] (F(\gamma \setminus x) - F(\gamma)), \\ (L_0^+ F)(\gamma) &= \int_{\mathbb{R}^d} dy \exp[-E^{\phi^+}(y, \gamma)] (F(\gamma \cup y) - F(\gamma)). \end{aligned}$$

**Theorem 1.** *Let  $k$  be the correlation functional of a probability measure  $\mu$  on  $(\Gamma, \mathcal{B}(\Gamma))$ , and let  $u$  be the corresponding Ursell functional. Assume that the following conditions are satisfied:*

- i)  $k$  fulfills the bound  $k(\eta) \leq (|\eta|!)^s C^{|\eta|}$ ,  $\eta \in \Gamma_0$ , for some  $0 \leq s < 1$  and  $C > 0$ . Here  $|\eta|$  denotes the cardinality of set  $\eta$ .
- ii)  $k$  is translation invariant.
- iii) The measure  $\mu$  has a decay of correlations in the sense that, for any  $n, m \in \mathbb{N}$ ,  $a \in \mathbb{R}^d$ ,  $a \neq 0$ , and  $\{x_1, \dots, x_{n+m}\} \in \Gamma^{(n+m)}$ ,

$$u(\{x_1, \dots, x_n, x_{n+1} + (a/\varepsilon), \dots, x_{n+m} + (a/\varepsilon)\}) \rightarrow 0 \quad \text{as } \varepsilon \rightarrow 0.$$

Then, for each  $\eta \in \Gamma_0$ ,

$$(\hat{L}_\varepsilon^- k)(\eta) \rightarrow c^-(k)(\hat{L}_0^- k)(\eta), \quad (\hat{L}_\varepsilon^+ k)(\eta) \rightarrow c^+(k)(\hat{L}_0^+ k)(\eta),$$

where

$$\begin{aligned} c^-(k) &:= \int_{\Gamma_0} \lambda(d\xi) e_\lambda(e^{-\phi^+} - 1, \xi) k(\xi), \\ c^+(k) &:= \int_{\Gamma_0} \lambda(d\xi) e_\lambda(e^{\phi^-} - 1, \xi) k(\xi \cup 0). \end{aligned} \tag{2}$$

*Proof.* A straightforward calculation (see [8]) shows that

$$\begin{aligned}
(\hat{L}_\varepsilon^{-*}k)(\eta) &= - \sum_{x \in \eta} \int_{\mathbb{R}^d} dy a_\varepsilon(x-y) r(x, y, \eta \setminus x) \\
&\quad \times \int_{\Gamma_0} \lambda(d\xi) k(\xi \cup \eta) e_\lambda(e^{\phi^-(x-\cdot) - \phi^+(y-\cdot)} - 1, \xi), \\
(\hat{L}_\varepsilon^{+*}k)(\eta) &= \sum_{y \in \eta} \int_{\mathbb{R}^d} dx a_\varepsilon(x-y) r(x, y, \eta \setminus y) \\
&\quad \times \int_{\Gamma_0} \lambda(d\xi) k(\xi \cup (\eta \setminus y) \cup x) e_\lambda(e^{\phi^-(x-\cdot) - \phi^+(y-\cdot)} - 1, \xi), \\
(\hat{L}_0^{-*}k)(\eta) &= - \sum_{x \in \eta} \exp[E^{\phi^-}(x, \eta \setminus x)] \\
&\quad \times \int_{\Gamma_0} \lambda(d\xi) e_\lambda(e^{\phi^-(x-\cdot)} - 1, \xi) k(\eta \cup \xi), \\
(\hat{L}_0^{+*}k)(\eta) &= \sum_{y \in \eta} \exp[-E^{\phi^-}(y, \eta \setminus y)] \\
&\quad \times \int_{\Gamma_0} \lambda(d\xi) e_\lambda(e^{-\phi^+(y-\cdot)} - 1, \xi) k((\eta \setminus y) \cup \xi).
\end{aligned} \tag{3}$$

We will now briefly explain the convergence of  $(\hat{L}_\varepsilon^{-*}k)(\eta)$  (the case of  $(\hat{L}_\varepsilon^{+*}k)(\eta)$  can be dealt with analogously). From (3) and the definition of  $\lambda$ , by making a change of variable, we easily have:

$$\begin{aligned}
(\hat{L}_\varepsilon^{-*}k)(\eta) &= - \sum_{x \in \eta} \int_{\mathbb{R}^d} dy a(y) r(x, (y/\varepsilon) + x, \eta \setminus x) \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^n \binom{n}{k} \\
&\quad \times \int_{(\mathbb{R}^d)^n} du_1 \cdots du_n \prod_{i=1}^k (e^{-\phi^+((y/\varepsilon)+x-u_i)} (e^{\phi^-(x-u_i)} - 1)) \\
&\quad \times \prod_{j=k+1}^n (e^{-\phi^+((y/\varepsilon)+x-u_j)} - 1) k(\xi \cup \{u_1, \dots, u_n\}) \\
&= - \sum_{x \in \eta} \int_{\mathbb{R}^d} dy a(y) r(x, (y/\varepsilon) + x, \eta \setminus x) \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{1}{k!(n-k)!} \\
&\quad \times \int_{(\mathbb{R}^d)^n} du_1 \cdots du_n \prod_{i=1}^k (e^{-\phi^+((y/\varepsilon)-u_i)} (e^{\phi^-(u_i)} - 1)) \prod_{j=k+1}^n (e^{-\phi^+(u_j)} - 1) \\
&\quad \times k(\xi \cup \{u_1 + x, \dots, u_k + x, u_{k+1} + x + (y/\varepsilon), \dots, u_n + x + (y/\varepsilon)\}).
\end{aligned}$$

Next, represent the correlation functionals in the above expression through a sum of Ursell functionals. Using the dominated convergence theorem and conditions i) and iii), we see that, in the limit, all the Ursell functionals containing at least one point from  $\xi \cup \{u_1 + x, \dots, u_k + x\}$  and at least one point from

$\{u_{k+1} + x + (y/\varepsilon), \dots, u_n + x + (y/\varepsilon)\}$  will vanish, and by virtue of ii), we conclude that  $(\hat{L}_\varepsilon^{-*}k)(\eta)$  converges to

$$\begin{aligned} & - \sum_{x \in \eta} \int_{\mathbb{R}^d} dy a(y) \exp[E^{\phi^-}(x, \eta \setminus x)] \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{1}{k!(n-k)!} \\ & \times \int_{(\mathbb{R}^d)^n} du_1 \dots du_n \prod_{i=1}^k (e^{\phi^-(x-u_i)} - 1) \prod_{j=k+1}^n (e^{-\phi^+(u_j)} - 1) \\ & \times k(\xi \cup \{u_1, \dots, u_k\}) k(\{u_{k+1}, \dots, u_n\}), \end{aligned}$$

from where the statement follows.  $\square$

From Theorem 1, we can make the following conclusion. Assume that a dynamics of hopping particle with Markov generator  $L_K$  has initial distribution  $\mu_0$ . Let  $\mu_t$  be the distribution of this process at time  $t > 0$ . Assume that, for each  $t \geq 0$ ,  $\mu_t$  has correlation functional  $k_t$  which satisfies conditions i)–iii) of Theorem 1. Further assume that  $c^\pm(k_t)$ ,  $t \geq 0$ , given through (2) remain constant. Then, we can expect that the scaled dynamics of hopping particles converges to a birth-and-death process with generator  $L_0 := c^-(k_0)L_0^- + c^+(k_0)L_0^+$  and initial distribution  $\mu_0$ . We will discuss below two cases where this statement can be proven rigorously (at least in the sense of convergence of the generators).

### 3 Convergence of non-equilibrium free dynamics

This case has been discussed in [14], so here we will explain its connection with Theorem 1.

Let  $\Theta \in \mathcal{B}(\Gamma)$  be the set of those configurations  $\gamma \in \Gamma$  for which there exist  $\alpha \geq d$  and  $K > 0$  such that

$$|\gamma \cap B(n)| \leq Kn^\alpha, \quad \text{for all } n \in \mathbb{N}, \quad (4)$$

where  $B(n)$  denotes the ball in  $\mathbb{R}^d$  centered at 0 and of radius  $n$ . Note that the estimate (4) controls the growth of the number of particles of  $\gamma$  at infinity.

Let  $a \in S(\mathbb{R}^d)$  (the Schwartz space of rapidly decreasing, infinitely differentiable functions on  $\mathbb{R}^d$ ). Consider a random walk in  $\mathbb{R}^d$  with transition kernel  $Q(x, dy) := a(x - y) dy$ . This is a Markov process in  $\mathbb{R}^d$  with generator

$$(L^{(1)}f)(x) = \int_{\mathbb{R}^d} (f(y) - f(x))a(x - y) dy.$$

The corresponding Markov semigroup on  $L^2(\mathbb{R}^d, dx)$  is then given by

$$(p_t f)(x) = e^{-t} f(x) + \int_{\mathbb{R}^d} G(x - y) f(y) dy, \quad (5)$$

where  $G$  is the inverse Fourier transform of  $e^{-t}(\exp[t(2\pi)^{d/2}\hat{a}] - 1)$ , where  $\hat{a}$  is the Fourier transform of  $a$ . (Note that we have normalized the direct and inverse

Fourier transforms so that they are unitary operators in  $L^2(\mathbb{R}^d \rightarrow \mathbb{C}, dx)$ .) For any  $\gamma \in \Theta$ , consider a dynamics of independent particles which starts at  $\gamma$  and such that each separate particle moves according to the semigroup  $p_t$  (i.e., independent random walks in  $\mathbb{R}^d$ ). Then, this process has *cádlág* paths on  $\Gamma$  and a.s. it never leaves  $\Theta$ , cf. [14]. The generator of the obtained Markov process on  $\Theta$  is then given by

$$(L_K F)(\gamma) = \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy a(x - y) (F(\gamma \setminus x \cup y) - F(\gamma)), \quad (6)$$

so that now  $\phi^\pm = 0$ .

**Proposition 1.** *Let  $\mu_0$  be a probability measure on  $\Gamma$  whose correlation functional  $k_0$  satisfies conditions i)–iii) of Theorem 1, and  $\mu_0(\Theta) = 1$ . Consider the Markov process on  $\Theta$  with the generator  $L_K$  given by (6) and with the initial distribution  $\mu_0$ . Denote by  $\mu_t$  the distribution of this process at time  $t > 0$ . Then, for each  $t > 0$ ,  $\mu_t$  has correlation functional  $k_t$  which satisfies conditions i)–iii) of Theorem 1, and furthermore  $c^-(k_t) = 1$  and  $c^+(k_t) = k_0^{(1)}$ ,  $t \geq 0$ .*

*Proof.* For each  $f \in C_0(\mathbb{R}^d)$  and  $t > 0$ , we have, by (1) and the construction of the process:

$$\begin{aligned} \int_{\Theta} \mu_t(d\gamma) e^{\langle f, \gamma \rangle} &= \int_{\Theta} \mu_0(d\gamma) \prod_{x \in \gamma} (p_t e^f)(x) \\ &= \int_{\Gamma_0} \lambda(d\eta) k_0(\eta) \prod_{x \in \eta} (p_t(e^f - 1))(x) \\ &= 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^d)^n} dx_1 \cdots dx_n k^{(n)}(x_1, \dots, x_n) \prod_{i=1}^n (p_t(e^f - 1))(x_i) \\ &= 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^d)^n} dx_1 \cdots dx_n (p_t^{\otimes n} k^{(n)})(x_1, \dots, x_n) \prod_{i=1}^n (e^{f(x_i)} - 1). \end{aligned}$$

Therefore,  $\mu_t$  has correlation functional  $k_t$ , and furthermore  $k_t^{(n)} = p_t^{\otimes n} k_0^{(n)}$ . The latter equality, in turn, implies that  $u_t^{(n)} = p_t^{\otimes n} u_0^{(n)}$ . From here it easily follows that, for each  $t > 0$ ,  $\mu_t$  satisfies assumptions i)–iii) of Theorem 1. Furthermore, by (2),

$$\begin{aligned} c^-(k_t) &= k_t(\emptyset) = 1, \\ c^+(k_t) &= k_t(\{0\}) = k_t^{(1)} = p_t k_0^{(1)} = k_0^{(1)}. \quad \square \end{aligned}$$

Thus, according to Section 2, we expect that the scaled free dynamics with initial distribution  $\mu_0$  converges to the birth-and-death process with generator

$$(L_0 F)(\gamma) = \sum_{x \in \gamma} (F(\gamma \setminus x) - F(\gamma)) + k_0^{(1)} \int_{\mathbb{R}^d} dy (F(\gamma \cup y) - F(\gamma)) \quad (7)$$

and initial distribution  $\mu_0$ . This dynamics can be constructed as follows, cf. [14, 19]. For each  $\gamma \in \Theta$ , denote by  $P_\gamma$  the law of a process on  $\Theta$  which is at  $\gamma$  at time zero, and after this, points of  $\gamma$  randomly die, independently of each other, so that the probability that at time  $t > 0$  a particle  $x \in \gamma$  is still alive is equal to  $e^{-t}$ . Next, let  $\pi$  denote the Poisson point process in  $\mathbb{R}^d \times (0, \infty)$  with the intensity measure  $k_0^{(1)} dx dt$ . The measure  $\pi$  is concentrated on configurations  $\hat{\gamma} = \{(x_n, t_n)\}_{n=1}^\infty$  in  $\mathbb{R}^d \times (0, \infty)$  such that  $\{x_n\}_{n=1}^\infty \in \Theta$ ,  $0 < t_1 < t_2 < \dots$ , and  $t_n \rightarrow \infty$  as  $n \rightarrow \infty$ . For any such configuration, we denote by  $P_{\hat{\gamma}}$  the law of a process on  $\Theta$  such that at time  $t = 0$ , the configuration is empty, and then at each time  $t_n$ ,  $n \in \mathbb{N}$ , a new particle is born at  $x_n$ , and after time  $t_n$  this particle randomly dies, independently of the other particles, so that at time  $s > t_n$  the probability that the particle is still alive is  $e^{-(s-t_n)}$ . Finally, the law of the process with generator (7) and initial distribution  $\mu_0$  is given by

$$\int \mu_0(d\gamma) P_\gamma * \int \pi(d\hat{\gamma}) P_{\hat{\gamma}}.$$

Here  $*$  stays for convolution of measures, see [14] for details.

We will use  $\ddot{\Gamma}$  to denote the space of multiple configurations over  $\mathbb{R}^d$  equipped with the vague topology, see e.g. [9] for details. Note that  $\Gamma \subset \ddot{\Gamma}$ , and the trace  $\sigma$ -algebra of  $\mathcal{B}(\ddot{\Gamma})$  on  $\Gamma$  is  $\mathcal{B}(\Gamma)$ .

**Theorem 2** ([14]). *Consider the stochastic process from Proposition 1 as taking values in  $\ddot{\Gamma}$ . Then, after scaling, this process converges, in the sense of weak convergence of finite-dimensional distributions, to the Markov process with the generator  $L_0$  given by (7) and with the initial distribution  $\mu_0$ .*

Note that the limiting process also lives in  $\Theta$ , and we used the  $\ddot{\Gamma}$  space only to identify the type of convergence.

For reader's convenience, let us explain the idea of the proof of Theorem 2. Fix arbitrary  $0 = t_0 < t_1 < t_2 < \dots < t_n$ ,  $n \in \mathbb{N}$ , and denote by  $\mu_{t_0, t_1, \dots, t_n}^\varepsilon$ ,  $\varepsilon \geq 0$ , the corresponding finite-dimensional distribution of the initial process scaled by  $\varepsilon > 0$ , and that of the limiting process if  $\varepsilon = 0$ , respectively. Then, by [9], the statement of the theorem is equivalent to stating that, for any non-positive  $f_0, f_1, \dots, f_n \in C_0(\mathbb{R}^d)$ ,

$$\begin{aligned} \int_{\Theta^n} \exp \left[ \sum_{i=0}^n \langle f_i, \gamma \rangle \right] d\mu_{t_0, t_1, \dots, t_n}^\varepsilon(\gamma_0, \gamma_1, \dots, \gamma_n) \\ \rightarrow \int_{\Theta^n} \exp \left[ \sum_{i=0}^n \langle f_i, \gamma \rangle \right] d\mu_{t_0, t_1, \dots, t_n}^0(\gamma_0, \gamma_1, \dots, \gamma_n) \quad \text{as } \varepsilon \rightarrow 0. \end{aligned} \quad (8)$$

For  $\varepsilon > 0$ , denote by  $p_t^\varepsilon(x, dy)$  the transition probability of the Markov semi-

group (4) scaled by  $\varepsilon$ . Set

$$g^\varepsilon(x) := e^{f_0(x)} \int_{\mathbb{R}^d} p_{t_1}^\varepsilon(x, dx_1) \int_{\mathbb{R}^d} p_{t_2-t_1}^\varepsilon(x_1, dx_2) \\ \times \cdots \times \int_{\mathbb{R}^d} p_{t_n-t_{n-1}}^\varepsilon(x_{n-1}, dx_n) \prod_{i=1}^n e^{f_i(x_i)}, \quad x \in \mathbb{R}^d.$$

Then, by (1) and the construction of the process, the first integral in (8) (with  $\varepsilon > 0$ ) is equal to

$$\int_{\Theta} \prod_{x \in \gamma} g^\varepsilon(x) \mu_0(d\gamma) \\ = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^d)^n} \prod_{i=1}^n (g^\varepsilon(x_i) - 1) k_0^{(n)}(x_1, \dots, x_n) dx_1 \cdots dx_n.$$

In the above integrals, one represents the correlation functions through the Ursell functions, makes a change of variables under the sign of integral, and after a careful analysis of the obtained expression, one takes its limit as  $\varepsilon \rightarrow 0$ . Finally, one shows that the obtained limit is indeed equal to the second integral in (8).

## 4 Convergence of equilibrium Kawasaki dynamics of interacting particles

In this section, we will consider equilibrium dynamics of interacting particles having a Gibbs measure as an equilibrium measure. Our result will extend that of [7], where just one special case of such a dynamics was considered (see also [15]). We start with a description of the class of Gibbs measures we are going to use.

A pair potential is a Borel-measurable function  $\phi : \mathbb{R}^d \rightarrow \mathbb{R} \cup \{+\infty\}$  such that  $\phi(-x) = \phi(x) \in \mathbb{R}$  for all  $x \in \mathbb{R}^d \setminus \{0\}$ . For  $\gamma \in \Gamma$  and  $x \in \mathbb{R}^d \setminus \gamma$ , we define a relative energy of interaction between a particle at  $x$  and the configuration  $\gamma$  as  $E(x, \gamma) := \sum_{y \in \gamma} \phi(x-y)$ , provided that the latter sum converges absolutely, and otherwise it is set to be  $= \infty$ . A (grand canonical) Gibbs measure corresponding to the pair potential  $\phi$  and activity  $z > 0$  is a probability measure  $\mu$  on  $(\Gamma, \mathcal{B}(\Gamma))$  which satisfies the Georgii–Nguyen–Zessin identity:

$$\int_{\Gamma} \mu(d\gamma) \sum_{x \in \gamma} F(\gamma, x) = \int_{\Gamma} \mu(d\gamma) \int_{\mathbb{R}^d} z dx \exp[-E(x, \gamma)] F(\gamma \cup x, x) \quad (9)$$

for any measurable function  $F : \Gamma \times \mathbb{R}^d \rightarrow [0, +\infty)$ . A pair potential  $\phi$  is said to be stable if there exists  $B \geq 0$  such that, for any  $\eta \in \Gamma_0$ ,

$$\sum_{\{x, y\} \subset \eta} \phi(x-y) \geq -B|\eta|. \quad (10)$$

In particular, we then have  $\phi(x) \geq -2B$ ,  $x \in \mathbb{R}^d$ . Next, we say that the condition of low activity–high temperature regime is fulfilled if

$$\int_{\mathbb{R}^d} |e^{-\phi(x)} - 1| z \, dx < (2e^{1+2B})^{-1}, \quad (11)$$

where  $B$  is as in (10). A classical result of Ruelle [17, 18] says that, under the assumption of stability and low activity–high temperature regime, there exists a Gibbs measure  $\mu$  corresponding to  $\phi$  and  $z$ , and this measure has correlation functional which satisfies conditions i)–iii) of Theorem 1, with  $s = 0$  in condition i) (which is then called the Ruelle bound). Furthermore, the corresponding Ursell functions satisfy  $u^{(n)}(0, \cdot, \dots, \cdot) \in L^1((\mathbb{R}^d)^{n-1}, dx_1 \cdots dx_n)$  for each  $n \geq 2$ . In what follows, we will assume that the potential  $\phi$  is also bounded from above outside some finite ball in  $\mathbb{R}^d$  (which is always true for any realistic potential, since it should converge to zero at infinity).

We now fix arbitrary parameters  $u, v \in [0, 1]$ , and assume that

$$\int_{\mathbb{R}^d} |\exp[(2(u \vee v) - 1)\phi(x)] - 1| \, dx < \infty. \quad (12)$$

It can be easily shown that, if  $u, v \in [0, 1/2]$ , then (12) is a corollary of (11) and the condition that  $\phi$  be bounded outside some finite ball. Note that, even if  $u \vee v \in (1/2, 1]$ , condition (12) still admits potentials which have ‘weak’ singularity at zero.

We introduce the set  $\mathcal{FC}_b(C_0(\mathbb{R}^d), \Gamma)$  of all functions of the form

$$\Gamma \ni \gamma \mapsto F(\gamma) = g(\langle f_1, \gamma \rangle, \dots, \langle f_N, \gamma \rangle),$$

where  $N \in \mathbb{N}$ ,  $f_1, \dots, f_N \in C_0(\mathbb{R}^d)$ , and  $g \in C_b(\mathbb{R}^N)$ . Here  $C_b(\mathbb{R}^N)$  denotes the set of all continuous bounded functions on  $\mathbb{R}^N$ . For each  $F \in \mathcal{FC}_b(C_0(\mathbb{R}^d), \Gamma)$ , we define

$$\begin{aligned} (L_K F)(\gamma) &= \frac{1}{2} \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy \, a(x - y) (\exp[uE(x, \gamma \setminus x) - (1 - v)E(y, \gamma \setminus x)] \\ &\quad + \exp[vE(x, \gamma \setminus x) - (1 - u)E(y, \gamma \setminus x)])(F(\gamma \setminus x \cup y) - F(\gamma)). \end{aligned} \quad (13)$$

Note that the first addend in (13) corresponds to the choice of  $\phi^- = u\phi$ ,  $\phi^+ = (1 - v)\phi$ , whereas the second addend corresponds to  $\phi^- = v\phi$ ,  $\phi^+ = (1 - u)\phi$ . In the special case where  $u = v$ , we get

$$\begin{aligned} (L_K F)(\gamma) &= \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy \, a(x - y) \exp[uE(x, \gamma \setminus x) - (1 - u)E(y, \gamma \setminus x)] \\ &\quad \times (F(\gamma \setminus x \cup y) - F(\gamma)). \end{aligned}$$

By [13],  $(L_K, \mathcal{FC}_b(C_0(\mathbb{R}^d), \Gamma))$  is a Hermitian, non-negative operator in  $L^2(\Gamma, \mu)$ , and we denote by  $(L_K, D(L_K))$  its Friedrichs’ extension. As shown in [13] by using the theory of Dirichlet forms, there exists a Markov process on  $\Gamma$  with *càdlàg*

paths whose generator is  $(L_K, D(L_K))$ . If we consider this process with initial distribution  $\mu$ , then it is an equilibrium process, i.e., it has distribution  $\mu_t = \mu$  at any moment of time  $t \geq 0$ . Thus, for each  $t \geq 0$ ,  $\mu_t = \mu$  has correlation function which satisfies conditions i)–iii) of Theorem 1.

**Lemma 1.** *Let  $k$  denote the correlation function of the Gibbs measure  $\mu$  under consideration. Denote*

$$C_u := \int_{\Gamma} \mu(d\gamma) \exp[-(1-u)\langle\phi, \gamma\rangle].$$

*Then we have:*

$$\int_{\Gamma_0} \lambda(d\xi) e_{\lambda}(e^{-(1-u)\phi} - 1, \xi) k(\xi) = C_u, \quad (14)$$

$$\int_{\Gamma_0} \lambda(d\xi) e_{\lambda}(e^{u\phi} - 1, \xi) k(\xi \cup 0) = z C_u. \quad (15)$$

*Proof.* Equality (14) follows from (1). Next, using (1), (9), and translation invariance of  $k$ , we have, for each  $f \in C_0(\mathbb{R}^d)$ :

$$\begin{aligned} & \int_{\mathbb{R}^d} dx f(x) \int_{\Gamma} \mu(d\gamma) \exp[-(1-u)\langle\phi, \gamma\rangle] \\ &= \int_{\mathbb{R}^d} dx f(x) \int_{\Gamma} \mu(d\gamma) \exp[-(1-u)E(x, \gamma)] \\ &= z^{-1} \int_{\Gamma} \mu(d\gamma) \sum_{x \in \gamma} f(x) \exp[uE(x, \gamma \setminus x)] \\ &= z^{-1} \int_{\Gamma} \mu(d\gamma) \sum_{x \in \gamma} f(x) \sum_{\xi \in \gamma \setminus x} e_{\lambda}(e^{u\phi} - 1, \xi) \\ &= z^{-1} \int_{\Gamma} \mu(d\gamma) \sum_{\xi \in \gamma} \sum_{x \in \xi} f(x) e_{\lambda}(e^{u\phi} - 1, \xi \setminus x) \\ &= z^{-1} \int_{\Gamma_0} \lambda(d\xi) k(\xi) \sum_{x \in \xi} f(x) e_{\lambda}(e^{u\phi} - 1, \xi \setminus x) \\ &= z^{-1} \int_{\Gamma_0} \lambda(d\xi) \int_{\mathbb{R}^d} dx k(\xi \cup x) f(x) e_{\lambda}(e^{u\phi} - 1, \xi) \\ &= z^{-1} \int_{\mathbb{R}^d} f(x) \int_{\Gamma_0} \lambda(d\xi) k(\xi_{-x} \cup 0) e_{\lambda}(e^{u\phi} - 1, \xi) \\ &= z^{-1} \int_{\mathbb{R}^d} f(x) \int_{\Gamma_0} \lambda(d\xi) k(\xi \cup 0) e_{\lambda}(e^{u\phi} - 1, \xi), \end{aligned}$$

from where equality (15) follows.  $\square$

Thus, by Lemma 1, according to Section 2, we expect that the scaled equilibrium dynamics (with initial distribution  $\mu$ ) converges to the birth-and-death

process with generator

$$\begin{aligned}
(L_0 F)(\gamma) &= \sum_{x \in \gamma} \frac{1}{2} (C_v \exp[uE(x, \gamma \setminus x)] + C_u \exp[vE(x, \gamma \setminus x)]) \\
&\quad \times (F(\gamma \setminus x) - F(\gamma)) \\
&+ \int_{\mathbb{R}^d} z dy \frac{1}{2} (C_u \exp[-(1-v)E(y, \gamma)] + C_v \exp[-(1-u)E(y, \gamma)]) \\
&\quad \times (F(\gamma \cup y) - F(\gamma)) \quad (16)
\end{aligned}$$

and the initial distribution  $\mu$ . In fact, by [13],  $(L_0, \mathcal{F}C_b(C_0(\mathbb{R}^d), \Gamma))$  is a Hermitian, non-negative operator in  $L^2(\Gamma, \mu)$ , and its Friedrichs' extension  $(L_0, D(L_0))$  is the generator of a Markov process on  $\Gamma$  with *càdlàg* paths.

We recall that  $L_\varepsilon$  denotes the  $L_K$  generator (given by (13)) scaled by  $\varepsilon$ . The following theorem states that, at least on an appropriate set of test functions, the operator  $L_\varepsilon$  converges to  $L_0$  in the  $L^2$ -norm.

**Theorem 3.** *For each  $f \in C_0(\mathbb{R}^d)$ , we have  $e^{\langle f, \cdot \rangle} \in D(L_\varepsilon)$  for all  $\varepsilon \geq 0$ , and*

$$L_\varepsilon e^{\langle f, \cdot \rangle} \rightarrow L_0 e^{\langle f, \cdot \rangle} \quad \text{in } L^2(\Gamma, \mu) \text{ as } \varepsilon \rightarrow 0.$$

*Proof.* We will only sketch the proof of the theorem. Let  $f \in C_0(\mathbb{R}^d)$ . By approximation, one easily shows that, for each  $\varepsilon \geq 0$ , the function  $F(\gamma) = e^{\langle f, \gamma \rangle}$  belongs to  $D(L_\varepsilon)$ , and that the action of  $L_\varepsilon$  onto  $F$  is given, for  $\varepsilon > 0$  by the right hand side of (13) in which  $a$  is replaced by  $a_\varepsilon$ , and for  $\varepsilon = 0$  by (16), respectively.

Denote

$$\begin{aligned}
(\mathcal{L}_\varepsilon^- F)(\gamma) &= \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy a_\varepsilon(x - y) \\
&\quad \times \exp[uE(x, \gamma \setminus x) - (1-v)E(y, \gamma \setminus x)] e^{\langle f, \gamma \setminus x \rangle} (1 - e^{f(x)}), \\
(\mathcal{L}_\varepsilon^+ F)(\gamma) &= \sum_{x \in \gamma} \int_{\mathbb{R}^d} dy a_\varepsilon(x - y) \\
&\quad \times \exp[uE(x, \gamma \setminus x) - (1-v)E(y, \gamma \setminus x)] e^{\langle f, \gamma \setminus x \rangle} (e^{f(y)} - 1), \\
(\mathcal{L}_0^- F)(\gamma) &= C_v \sum_{x \in \gamma} \exp[uE(x, \gamma \setminus x)] e^{\langle f, \gamma \setminus x \rangle} (1 - e^{f(x)}), \\
(\mathcal{L}_0^+ F)(\gamma) &= C_u z \int_{\mathbb{R}^d} dy \exp[-(1-v)E(y, \gamma)] e^{\langle f, \gamma \rangle} (e^{f(y)} - 1).
\end{aligned}$$

To prove the theorem, it suffices to show that

$$\begin{aligned}
\|\mathcal{L}_\varepsilon^\pm F\|_{L^2(\Gamma, \mu)}^2 &\rightarrow \|\mathcal{L}_0^\pm F\|_{L^2(\Gamma, \mu)}^2, \\
(\mathcal{L}_\varepsilon^\pm F, \mathcal{L}_0^\pm F)_{L^2(\Gamma, \mu)} &\rightarrow \|\mathcal{L}_0^\pm F\|_{L^2(\Gamma, \mu)}^2 \quad (17)
\end{aligned}$$

as  $\varepsilon \rightarrow 0$ . To this end, one proceeds as follows. By using (9), one represents each of the expressions appearing in (17) in terms of integrals over  $\Gamma$  with

respect to  $\mu$ , as well as integrals over  $\mathbb{R}^d$  with respect to Lebesgue measure. As a result one gets rid of all summations  $\sum_{x \in \gamma}$ . Then, one makes a change of variables, so that instead of  $a_\varepsilon(x - y)$  one gets  $a(x)$ , and in  $y$  variable one gets a function which is dominated by an integrable function of  $y$ . Next, one replaces integration  $\int_\Gamma \mu(d\gamma) \cdots$  by corresponding integration  $\int_{\Gamma_0} \lambda(d\eta) k(\eta) \cdots$ . In the obtained expression, one represents the correlation functional through a sum of Ursell functionals. Finally, one takes the limit as  $\varepsilon \rightarrow 0$  by analogy with the final part of the proof of Theorem 1.  $\square$

By using the well-known result of the theory of semigroups (see e.g. [2]), we get the following corollary of Theorem 3.

**Corollary 1.** *Assume that the set of finite linear combinations of exponential functions  $e^{(f, \cdot)}$ ,  $f \in C_0(\mathbb{R}^d)$ , is a core for the limiting generator  $(L_0, D(L_0))$ . Then, we have the weak convergence of finite-dimensional distributions of the scaled Markov process in  $\Gamma$  with the generator  $(L_\varepsilon, D(L_\varepsilon))$  and with the initial distribution  $\mu$  to the Markov process in  $\Gamma$  with the generator  $(L_0, D(L_0))$  and with the initial distribution  $\mu$ . In particular, if additionally  $\phi \geq 0$ , then this kind of convergence holds when  $u = v = 0$ .*

We note that the final statement of Corollary 1 holds due to a result of [12] on essential self-adjointness of the generator of Glauber dynamics in the case  $\phi \geq 0$  and  $u = v = 0$  (see also [7]). In the latter case, we even expect that the *weak convergence of laws* holds. To this end, one needs to consider all processes as taking values in a negative Sobolev space. The tightness of the laws of scaled processes may be proven by analogy with the proof of [5, Theorem 7.1]. Next, one shows that this set of laws has, in fact, a unique limiting point—the law of the Markov process with generator  $(L_0, D(L_0))$  and initial distribution  $\mu_0$ . This is done by identifying the limit via the martingale problem, and using convergence of the generators (compare with the proof of [6, Theorem 6.7] and that of [5, Theorem 7.5]).

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