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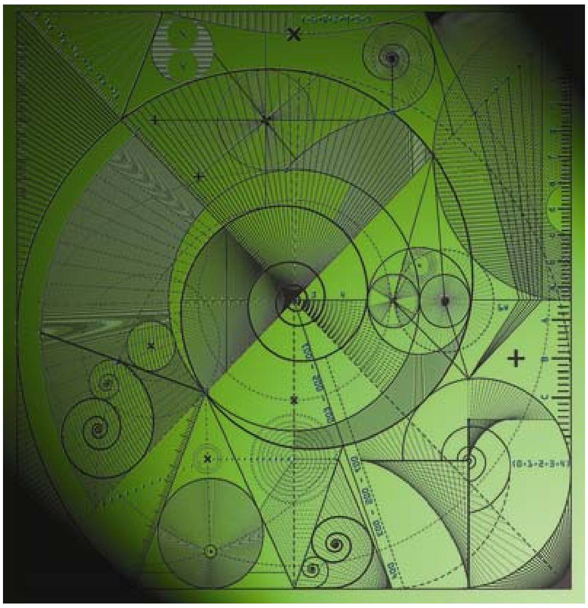

Data assimilation for state and parameter estimation: application to morphodynamic modelling

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Abstract

Data assimilation is predominantly used for state estimation; combining observational data with model predictions to produce an updated model state that most accurately approximates the true system state whilst keeping the model parameters fixed. This updated model state is then used to initiate the next model forecast. Even with perfect initial data, inaccurate model parameters will lead to the growth of prediction errors. To generate reliable forecasts we need good estimates of both the current system state and the model parameters. This paper presents research into data assimilation methods for morphodynamic model state and parameter estimation. First, we focus on state estimation and describe implementation of a three dimensional variational (3D-Var) data assimilation scheme in a simple 2D morphodynamic model of Morecambe Bay, UK. The assimilation of observations of bathymetry derived from SAR satellite imagery and a ship-borne survey is shown to significantly improve the predictive capability of the model over a 2 year run. Here, the model parameters are set by manual calibration; this is laborious and is found to produce different parameter values depending on the type and coverage of the validation dataset. The second part of this paper considers the problem of model parameter estimation in more detail. We explain how, by employing the technique of state augmentation, it is possible to use data assimilation to estimate uncertain model parameters concurrently with the model state. This approach removes inefficiencies associated with manual calibration and enables more effective use of observational data. We outline the development of a novel hybrid sequential 3D-Var data assimilation algorithm for joint state-parameter estimation and demonstrate its efficacy using an idealised 1D sediment transport model. The results of this study are extremely positive and suggest that there is great potential for the use of data assimilation based state-parameter estimation in coastal morphodynamic modelling.

1 Introduction

An increase in extreme and hazardous weather events in recent years has led to growing concern over the effects of global climate change. Expected sea level rise combined with an increase in the frequency and intensity of storm events has profound implications for coastal regions and highlights the need for greater knowledge and understanding of how the morphology of the coastal zone is changing (Nicholls et al., 2007).

Morphodynamic change can have wide reaching environmental, human social and economic impacts. Accurate knowledge of coastal morphology is fundamental to effective shoreline management and protection; for example, managing coastal erosion, assessing the potential impact of human use of coastal land, monitoring wildlife habitats and the mitigation of flood hazard. It is a complex and challenging subject but one which is of great practical importance.

Unfortunately, knowledge of evolving near-shore bathymetry is limited. In many regions, the topography of the sea bed can change rapidly over time. Obtaining full bathymetric surveys can be time consuming and expensive and so it is generally not feasible to collect observational data at the spatial and temporal frequency required to track these changes effectively. Consequently, bottom topography is a large source of uncertainty in coastal inundation modelling and this can strongly influence the quality of model predictions (Brown et al., 2007; Hesselink et al., 2003).

Computational morphodynamic models are becoming increasingly sophisticated in an attempt to compensate for this lack of complete data (e.g. Lesser et al., 2004). Modelling the continual interaction between water flow and bathymetry in the coastal zone presents a significant challenge. In addition to difficulties capturing the underlying physics, models are limited by imperfect knowledge of initial conditions and parameters. Even with a perfect model, uncertainties in initial conditions and parameters will lead to the growth of forecast error and therefore affect the ability of a coastal morphodynamic model to accurately predict the true state of the near-shore environment. In order to generate reliable forecasts of long term morphodynamic evolution we need to have good estimates of both the model parameters and the current bathymetry.

Data assimilation is a mathematical technique that enables the optimal use of model and observational resources and offers the potential to generate forecasts that are more accurate than using a model alone. It is most commonly used to produce initial conditions for state estimation: estimating model variables whilst keeping the model parameters fixed. However, it is also possible to use data assimilation to provide estimates of uncertain model parameters. Data assimilation techniques have been employed in the context of atmospheric and oceanic prediction for some years but have only relatively recently begun to be explored in the context of morphodynamic modelling. In this paper we present research into data assimilation methods for morphodynamic model state and parameter estimation conducted as part of the Natural Environment Research Council (NERC) funded Flood Risk from Extreme Events (FREE) programme. The project extends a previous feasibility study by Scott and Mason (2007) in which a 2D horizontal (2DH) decoupled morphodynamic model of Morecambe Bay, UK was enhanced through the assimilation of partial observations in the form of waterlines derived from synthetic aperture radar (SAR) satellite images (Mason et al., 2001) using a simple optimal interpolation (OI) algorithm. Despite the known deficiencies of the OI method (see, e.g. Lorenc, 1981) the data assimilation was shown to improve the ability of the model to predict large scale changes in bathymetry in the bay over a 3 year period.

This paper has two main parts, each describing an independent but complementary aspect of the project. The first focuses on the state estimation problem and describes implementation of a more robust three dimensional variational (3D-Var) data assimilation scheme for Morecambe Bay. The bathymetric component of the model models the tidal flow and the sediment transport to predict changes in the bathymetry of the bay. The sediment transport rate is computed using a semi-empirical formula containing two parameters that must be calibrated to fit the physical characteristics of the specific study site and the particular conditions being modelled. Here, the parameters are determined by manually calibrating the model against a set of validation observations; the model is run with various parameter combinations and the fit to observations assessed using Brier Skill scores.

The second part of this paper addresses the problem of parameter estimation in morphodynamic modelling in more detail and explains how data assimilation can be used to estimate uncertain model parameters concurrently with the model state. In this case the model parameters and predicted model state are updated simultaneously, rather than being treated as two individual processes, thus removing inefficiencies associated with manual calibration and making better use of the available observational data. Data assimilation techniques have the further advantage over many other parameter estimation methods in that they offer a framework for explicitly accounting for all sources of uncertainty. Supplementary material and discussions of the work presented in this section can be found in Smith et al. (2009a,b); Smith (2010) and Smith et al. (2011).

We begin in section 2 with a brief summary of some of the various different approaches to model calibration in coastal modelling. In section 3 we describe the theoretical formulation of the data assimilation problem for state estimation in a general system model. We introduce the terminology and notation that we will use throughout this paper and outline the 3D-Var framework used in this work. Section 4 introduces the equations upon which the sediment transport models considered this study are based. Section 5 gives an overview of the Morecambe Bay model and describes how the assimilation scheme developed by Scott and Mason (2007) was improved by replacing the original OI method with a 3D-Var algorithm in a study using data covering the period 2003 to 2005. We give details of the approach used to calibrate the parameters appearing in the sediment transport equation, discuss its limitations, and assess the results. In section 6 we explore the potential for joint state-parameter estimation in morphodynamic modelling. We outline the development of a new hybrid 3D-Var data assimilation algorithm that enables us to estimate uncertain model parameters alongside the model state variables as part of the assimilation process. Application of the technique is demonstrated using an idealised 1D sediment transport model. Conclusions and potential future developments are discussed in section 7.

2 Model calibration in coastal modelling

Parameter estimation is a fundamental part of the development of a morphodynamic model. Parameters are typically used as a way of representing processes that are not completely known or understood, or where limitations to computer power constrain the model resolution and therefore the level of detail that can be described. Lack of detail in our knowledge of the various processes governing sediment transport means that sediment transport models usually contain empirical or heuristic elements derived from practical experience rather than physical laws. A consequence of this is that these models will often contain parameters that are not directly measurable and which must be 'tuned' in order to calibrate the model to a specific field site.

Poorly known parameters are a key source of uncertainty in sediment transport models (Soulsby, 1997). Even if a model is perfect and the initial bathymetry well defined, errors in the parameters will affect the accuracy of the sediment transport flux calculation, leading to the growth of forecast error and in turn the accuracy of the predicted bed level changes. In most cases, parameter estimation is addressed as a separate issue to state estimation and model calibration is performed offline in a separate calculation. The classic approach is manual calibration or tuning of the model against observational data, as is described in section 5. However, the increase in computational capabilities in recent years has seen the development of many new, often complex, automated parameter optimisation algorithms. A variety of schemes are presented in the coastal modelling literature. Some, such as the downhill simplex optimisation (Hill et al., 2003), genetic algorithm (Knaapen and Hulscher, 2003), and hybrid genetic algorithm (Ruessink, 2005a), are based on determining a single best parameter set, whereas probabilistic approaches, such as classical Bayesian (Wüst, 2004) and Bayesian Generalised Likelihood Uncertainty Estimation (GLUE) (Beven and Binley, 1992; Ruessink, 2005b, 2006), are based on the principle that there is no single best parameter set. Instead, the parameters are treated as probabilistic variables with each parameter set being assigned a likelihood value.

In terms of data requirements, set-up and computational costs, adaptability, ease of implementation, etc. each of these methodologies has different strengths and weaknesses. Often it will be the chosen application that will make certain methods more appropriate than others; the suitability of a particular approach will depend on factors such as model complexity, availability of observational data, computational resources and user expertise. In section 6, we present a novel approach to model parameter estimation using a hybrid 3D-Var data assimilation scheme.

3 Data assimilation

There are many different types of data assimilation algorithm, each varying in formulation, complexity, optimality and suitability for practical application. A useful overview of some of the most common data assimilation methods used in meteorology and oceanography are given in the review articles Ehrendorfer (2007); Ghil and Malanotte-Rizzoli (1991) and Nichols (2009). More detailed mathematical formulations can be found in texts such as Daley, 1991; Kalnay, 2003, and Lewis et al., 2006.

One of the main objectives of this project was to demonstrate the utility of data assimilation for morphodynamic prediction, rather than to develop an operational assimilation-forecast system. It was important to use a stable and well established method. 3D-Var is a popular choice for state estimation in large problems; the approach has many advantages, such as ease of implementation (no model adjoints required), numerical robustness and computational efficiency. Although standard 3D-Var is designed to produce an analysis at a single time, by applying the method sequentially using a cyclic assimilation-forecast approach we can utilise time series of observational data. Sequential 3D-Var is the primary technique used in the work presented here.

Since the 3D-Var method is applicable to a wide range of contexts we formulate the assimilation problem for state estimation in a general system model. The model equations used in this study are introduced in section 4 with further details given in sections 5 and 6. The extension of the method to combined state-parameter estimation is given in section 6.1. Our notation broadly follows that of Ide et al., 1997.

We consider the discrete, non-linear, time invariant dynamical system model

$$\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k, \mathbf{p}) \qquad k = 0, 1, \ldots \tag{1}$$

where $\mathbf{z}_k \in \mathbb{R}^m$ is the model state vector at time t_k. The operator $\mathbf{f} : \mathbb{R}^m \longrightarrow \mathbb{R}^m$ is a non-linear function describing the evolution of the state from time t_k to t_{k+1}. and $\mathbf{p} \in \mathbb{R}^q$ is a vector of q (uncertain) model parameters. We assume that specification of the model state and parameters at time t_k uniquely determines the model state at all future times.

For sequential assimilation, we start at a given initial time t_k with an *a priori* or model background state $\mathbf{z}_k^b \in \mathbb{R}^m$, with error $\boldsymbol{\varepsilon}_{\mathbf{z}_k}^b \in \mathbb{R}^m$. This should be a best guess estimate of the current true dynamical system state and is typically taken from a previous model forecast.

The available observations at time t_k are represented by the vector $\mathbf{y}_k \in \mathbb{R}^{r_k}$ and can be related to the model state by the equations

$$\mathbf{y}_k = \mathbf{h}(\mathbf{z}_k) + \boldsymbol{\delta}_k, \qquad k = 0, 1, \ldots \tag{2}$$

The vector $\boldsymbol{\delta}_k \in \mathbb{R}^{r_k}$ represents the observation errors. These errors are commonly assumed to be unbiased, serially uncorrelated, stochastic variables, with a given probability distribution (Lewis et al., 2006). Note that the number of observations r_k may vary with time and will generally only provide partial coverage of the model domain. The operator $\mathbf{h} : \mathbb{R}^m \longrightarrow \mathbb{R}^{r_k}$ is an observation operator that maps from model to observation space; it acts to convert the model state variables to model values of the observed variable and is in general nonlinear.

In many practical applications observations are not taken simultaneously but are collected across a given time window. The convention in 3D-Var is to assume that the state does not evolve significantly within this period and treat all observations as if they had been taken at the same time. This time is typically set to be the midpoint of the observation time window.

The aim of data assimilation for state estimation is to combine the dynamical system model background state $\mathbf{z}_k^b$ with the observations $\mathbf{y}_k$ to produce an updated model state that gives the best estimate of the true system state $\mathbf{z}_k^t$ at a given time t_k. This optimal estimate is called the *analysis* and is denoted $\mathbf{z}_k^a$. The model parameters $\mathbf{p}$ are assumed to be known constant values.

In 3D-Var the analysis is found by minimising a cost function measuring the distance between the current model state and the background and observations

$$\begin{aligned} J(\mathbf{z}_k) &= (\mathbf{z}_k - \mathbf{z}_k^b)^T {\mathbf{B}_k}^{-1} (\mathbf{z}_k - \mathbf{z}_k^b) \\ &+ (\mathbf{y}_k - \mathbf{h}_k(\mathbf{z}_k))^T \mathbf{R}_k^{-1} (\mathbf{y}_k - \mathbf{h}_k(\mathbf{z}_k)). \end{aligned} \tag{3}$$

The weighting matrices $\mathbf{B}_k \in \mathbb{R}^{m \times m}$ and $\mathbf{R}_k \in \mathbb{R}^{r_k \times r_k}$ are the covariance matrices of the background and observations errors. These matrices represent our prior assumptions about the errors associated with the background and observations and determine the relative weighting given to $\mathbf{z}_k^b$ and $\mathbf{y}_k$ in the analysis. The appropriate specification of these matrices is central to the success of the assimilation scheme.

Observation errors originate from instrumental error, errors in the operator $\mathbf{h}$ and representativeness errors (observing scales that cannot be represented in the model (Daley, 1991)). It is common practice to assume that the errors for each observation type are statistically stationary and temporally and spatially uncorrelated. $\mathbf{R}$ is then taken to be a diagonal matrix with the observation error variances along the main diagonal.

Prescription of the matrix $\mathbf{B}_k$ offers a significant challenge (Bannister, 2008a,b). The standard approach in 3D-Var is to assume that the errors in the background state are approximately statistically stationary. The background error covariances can then be approximated by a fixed matrix (i.e. $\mathbf{B}_k = \mathbf{B}$, for all k). This assumption makes 3D-Var a particularly attractive option for large scale systems. We can write $\mathbf{B}$ as the product of the background error variances and correlations, i.e. $\mathbf{B} = \boldsymbol{\sigma}\boldsymbol{\rho}\boldsymbol{\sigma}$ where $\boldsymbol{\sigma}$ is a diagonal matrix of estimated background error standard deviations and $\boldsymbol{\rho}$ is a symmetric matrix of background error correlations. The data assimilation problem can be further simplified by assuming that these error correlations are homogeneous and isotropic. The matrix $\boldsymbol{\rho}$ can then be modelled using an analytic correlation function (see e.g. Daley (1991) for further discussion on this). In the idealised 1D model experiments described in section 6 we use a simple covariance model for $\mathbf{B}$ based on a first order auto-regressive function (Rodgers, 2000). A particular benefit of this model is that the matrix inverse can be calculated explicitly and has a particularly simple tri-diagonal form.

The matrix $\mathbf{B}$ has $(m \times m)$ elements; for large scale systems, such as the Morecambe Bay model, it will be extremely large and, in general, impractical if not impossible to explicitly compute and invert. For the work described in section 5 the inverse of $\mathbf{B}$ is specified directly using a Laplace based correlation model that gives similar results to a Gaussian correlation function (full details are given in Thornhill et al., 2011).

The 3D-Var method solves the nonlinear optimization problem (3) numerically, using a gradient based descent algorithm to iterate to the minimising solution. For the work described here the scheme is applied cyclically as part of an assimilation-forecast algorithm. The model is evolved one step at a time, assimilating the observations in the order that they become available. At each new assimilation time the current observational data are combined with the current model forecast (background) state and the analysis found through minimisation of (3). This updated model state estimate is then integrated forward using the model (1) to become the background state at the next assimilation time and the minimisation process is repeated.

4 The sediment transport model

The bathymetric component of a morphodynamic model describes changes in bathymetry due to the transport of sediment by the water flow regime under consideration. The sediment transport rate is a complex function of the water properties plus characteristics of the sediment such as density and grain size (Long et al., 2008). There is no universally agreed method for computing the sediment transport; numerous different formulae have been proposed, many of which are presented in Soulsby (1997) and van Rijn (1993). The work described in this paper uses the power law equation

$$q = A|u|^n, \tag{4}$$

where q is the sediment transport rate in the direction of the depth averaged current, $u = (u_x, u_y)$, and A and n are parameters whose values need to be set. This equation is one of the most basic sediment transport flux formulae and approximates the combined bed-load and suspended load sediment transport rate. It is based on a simplification of an equation formulated by van Rijn (1993) and has been chosen for its suitability to our joint state-parameter estimation work.

The parameter A is a dimensional constant whose value depends on various properties of the sediment and water, such as flow depth and velocity range, sediment grain size and kinematic viscosity (Soulsby, 1997). The derivation of the parameter n is less clear but it generally takes a value in the range $1 \leq n \leq 4$. Typically A and n are specified by determining theoretical values, or by calibrating the model against observations. The optimal values can vary from site to site and calibration of the model is generally carried out by running the model with different combinations of the parameters and then comparing the results with available observations to find the set that yield the best results. In regions where the maximum depth averaged current is around $1.0\,\mathrm{ms}^{-1}$, the sediment transport rate is not strongly dependent on n. In the Morecambe Bay model this occurs on the tidal flats. In general, adjusting A will affect the accretion and erosion over the whole model domain, whereas adjusting n will affect the volumes in the tidal channels only. The effect of increasing n for higher values of A can be seen in figure 1 where the value of the sediment transport rate is plotted for different combinations of A and n using a value of $u = 2.0\,\mathrm{ms}^{-1}$, representing the maximum velocity in the tidal channels.

The bathymetry is updated by solving the sediment transport equation

$$\frac{\partial z}{\partial t} = \frac{-1}{1-\epsilon}\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}\right), \tag{5}$$

where z is the bathymetry, t is time, q_x and q_y are the x and y components of the sediment transport rate, and ϵ is the sediment porosity (Soulsby, 1997). The sediment porosity is a non-dimensional value, expressed as a fraction between 0 and 1, that depends on the degree of sediment sorting and compaction. Its value can be obtained using the measurement techniques described in Soulsby (1997). In the absence

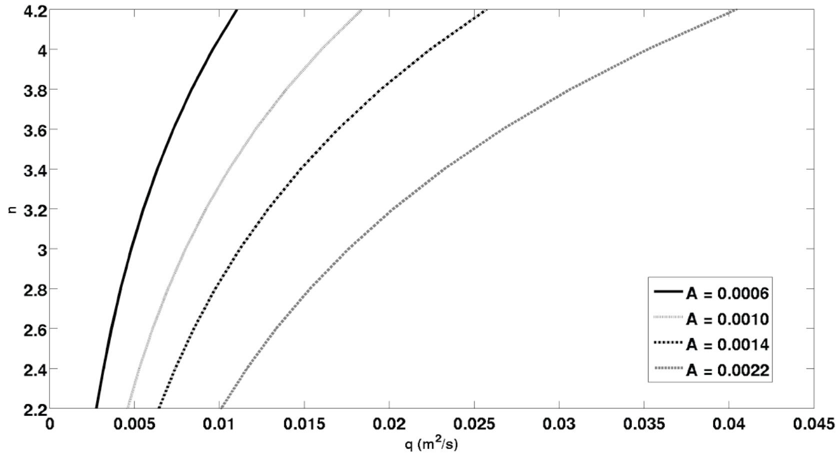

Figure 1: The effect of changing values of the parameters A and n in the sediment transport equation on the sediment transport rate, q, for a current velocity $u = 2.0\,\mathrm{ms}^{-1}$.

of any information on sediment characteristics a default value of $\epsilon = 0.4$ is recommended; this corresponds to a natural sand bed with average sorting and packing.

The sediment transport models employed in sections 5 and 6 below each take a different approach to solving equation (5). A full description of the numerical model implemented for the Morecambe Bay study site is given in Scott and Mason (2007). Details of the idealised 1D model used in the development of the joint state-parameter data assimilation scheme are given in section 6.3.

5 Morphodynamic model for Morecambe Bay

5.1 Morecambe Bay study site

Morecambe Bay is a large macrotidal bay in the north-west of Britain covering 340 km^2 (figure 2). It is comprised of large sub-tidal channels and inter-tidal sandbanks that can change position over a short period of time. The normal spring tidal range in the bay is 8.2 m, although due to the size of the bay this range is variable with location. The semi-diurnal ebb and flood tides are of different durations, with the ebb tide running around 40 minutes longer than the flood tide at Heysham. The sediments in the bay vary from silts in the inner bay, to very fine and fine sand in the inter-tidal regions, and coarser gravels near the mouth of the bay (Shoreline Management Partnership (SMP), 1996). During storm surges waves propagating up the channels leave some coastal areas vulnerable to flooding, and understanding the position of the channels and their evolution over time is essential for correctly assessing which areas most at risk.

5.2 Hydrodynamic and sediment transport model

The morphodynamic model of Morecambe Bay uses a 2DH (depth-averaged) hydrodynamic model for the tidal flows, but excludes waves as the tidal currents are the predominant factor in sediment transport in the bay (Mason et al., 1999). The bathymetric component of the model combines the sediment conservation equation (5) with the equation describing the sediment transport process (4). The model uses a grid of 176×196 cells, with a resolution of 240 m for both the hydrodynamic and bathymetric components. A full description of the model implementation is given in Scott and Mason (2007).

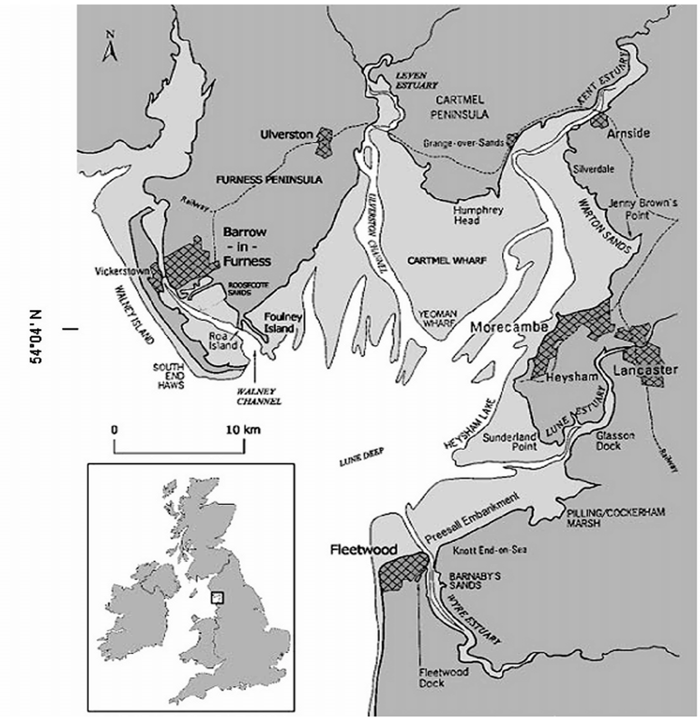

Figure 2: The main geographic features of Morecambe Bay, indicating main place names and settlements (based on O.S. maps (revised 1968-71)).

5.3 Implementation of 3D-Var data assimilation

The state vector in this implementation of the 3D-Var scheme is the bathymetry of the bay, effectively a vector of bed heights representing the 2D model of the seabed. The decoupled hydrodynamic and bathymetric model components constitute the system model (1) which describes the evolution of the bathymetry as a function of time. The observations were a series of waterlines derived from the SAR imagery; these provided a series of partial observations of the bathymetry of the bay over a period of almost 3 years from January 2003 to November 2005. Waterlines are effectively single quasi-contours across the bay marking the boundary between sea and land at the tide and wind conditions prevailing when the satellite observations were taken. The water height relative to the datum along the boundary is found using the hydrodynamic model of the bay for the time of the satellite observation. The water depth along this boundary is by definition zero, and as the bathymetry equals the water height minus the water depth, the bathymetry must equal the water height along this boundary line. In addition to the waterline data, we had a ship-borne survey of the deeper areas of the bay (referred to as the swath data) from February 2005. The construction of the error covariances for the background estimate of the bathymetry directly specifies the inverse error covariance matrix $\mathbf{B}^{-1}$ using a Laplacian correlation model. This approximation provides a computationally tractable matrix (Johnson, 2003; Johnson et al., 2005). The observation operator was simply an interpolation from the model points to the observation points, and the error covariance matrix was assumed to be a simple diagonal matrix. This implies that there are no correlations between the observation errors, which is a reasonable assumption in our case, and allows for the efficient computation of the cost function.

A full description and discussion of the implementation of the 3D-Var scheme for the Morecambe Bay model is described in Thornhill et al. (2011). The model and assimilation system was run using an initial bathymetry derived from earlier work (Mason et al., 1999) and updated using waterlines covering the period from January 2003 to December 2004, providing a more realistic initial bathymetry for the subsequent model runs. The model was then run from January 2004 to November 2005, assimilating 12 waterlines covering this period and the ship-borne survey from February 2005. We also investigated the impact of combining the swath data with the waterline data derived from the SAR images. The results from this work showed a substantial improvement in the predicted bathymetry when the data assimilation

A	n					
	2.2	**2.6**	**3.0**	**3.4**	**3.8**	**4.2**
0.0006		0.17	0.18	0.21	0.24	0.27
0.0012	0.14	0.21	0.27	0.32	0.34	0.36
0.0018	0.19	0.31	0.34	0.39	0.35	
0.0020	0.22	0.33	0.36	0.35		

Table 1: Brier skill scores obtained from swath data for different combinations of parameters A and n.

was applied.

5.4 Calibration of the Morecambe Bay model

The parameters A and n in the sediment transport flux formula (4) need to be set; this can be done by determining theoretical values, or by calibrating the model against observations. The focus of this part of the project was to demonstrate the utility of the 3D-Var state estimation assimilation scheme for the Morecambe Bay model as a first step towards the implementation of a joint state-parameter estimation algorithm; the current model was therefore tuned using a traditional calibration approach. This entailed running the model with different combinations of the parameters, and comparing the results with observations to find the set that yield the best results. The calibration covered the range 0.006 to $0.02\,\mathrm{ms}^{-1}$ for A and 2.2 to 4.2 for n. The comparison with observations was done by running the model to a point just before the swath data was taken and comparing the output of the assimilation system with this data. The measure used to determine the optimal combination of parameters was the Brier Skill score, defined as

$$\text{Brier Skill Score} = 1 - \frac{\left\langle (b_m - b_o)^2 \right\rangle}{\left\langle (b_i - b_o)^2 \right\rangle}, \tag{6}$$

where b_o is a vector containing the observations of bathymetry, b_m is a vector containing the modelled bathymetries for each observation in b_o, b_i is a vector containing initial bathymetries for each b_o and $< \cdot >$ defines the average over all values. This is a widely used score for assessing the skill of a model in accurately predicting the state of a system and provides a comparative measure for the outcomes of the different parameter sets (Sutherland et al., 2004).

The results for this calibration are shown in table 5.4. The Brier skill scores ranged from 0.14 to 0.39 with the highest score obtained for parameter values $A = 0.0018\,\mathrm{ms}^{-1}$ and $n = 3.4$.

The final results for the model are compared with a LiDAR survey of the inter-tidal areas of the bay which was obtained in November 2005. Comparison with this data for model runs with the best and worst scoring parameter combinations gives an indication of the impact of the chosen values on the predicted bathymetry (figure 3). In this figure, (a) shows the predicted bathymetry for values of $A = 0.0018\,\mathrm{ms}^{-1}$, $n = 3.4$ (highest skill score), (b) shows the LiDAR data resampled to the model grid size, and (c) shows the predicted bathymetry for values of $A = 0.0012\,\mathrm{ms}^{-1}$, $n = 2.2$ (lowest skill score). The differences between the two model predictions show the influence of the parameters. The larger values of A and n produce deeper channels, and the depositional areas (lighter greys) are a better match to the LiDAR data. The lower values of A and n (figure 3c) shows lower erosion in the channels, which matches the LiDAR data somewhat better, but does not show as good a match for the depositional areas. However, the overall difference between the bathymetry predicted for the best and worst values for the parameters is not dramatic. It is worth noting that the swath data used for the calibration was from a ship-borne survey, and therefore did not cover the shallower parts of the bay. This may have introduced a bias in that the calibration would be more suited to the deeper channels of the bay and may be less appropriate for the tidal flats and the shallower channels. If we examine the Brier skill scores calculated using the LiDAR data (which covers the entire bay) we find that the optimum parameter values are somewhat different from those obtained from the initial calibration. As table 5.4 shows, the highest score is now obtained when $A = 0.0012\,\mathrm{ms}^{-1}$ and $n = 3.0$. This difference could be due to the 9 month difference

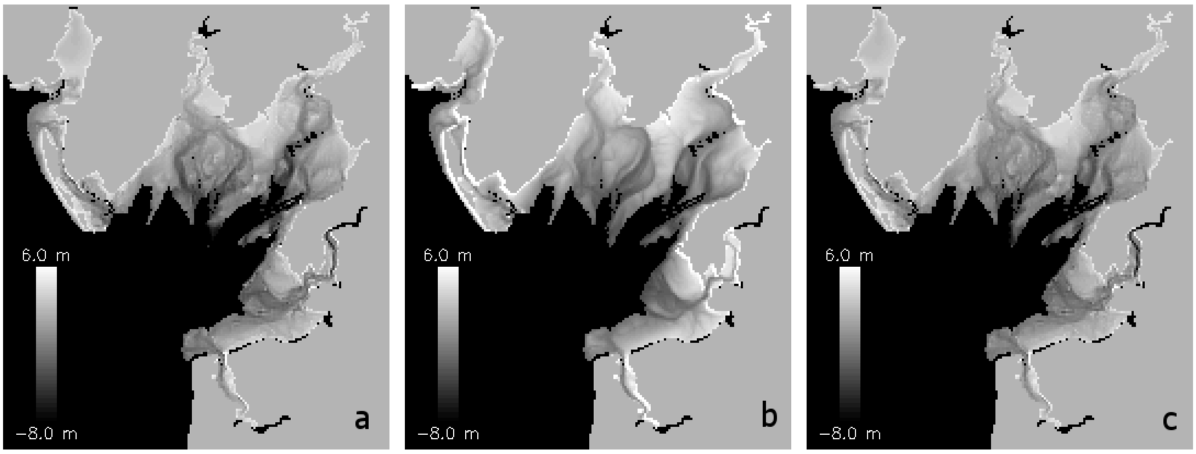

Figure 3: The bathymetry of Morecambe Bay for November 2005: (a) Predictions of the assimilation system for using $A = 0.0018$, $n = 3.4$. (b) LiDAR data plotted at the same resolution as the model. (c) Predictions of the assimilation system for $A = 0.0012$, $n = 2.2$.

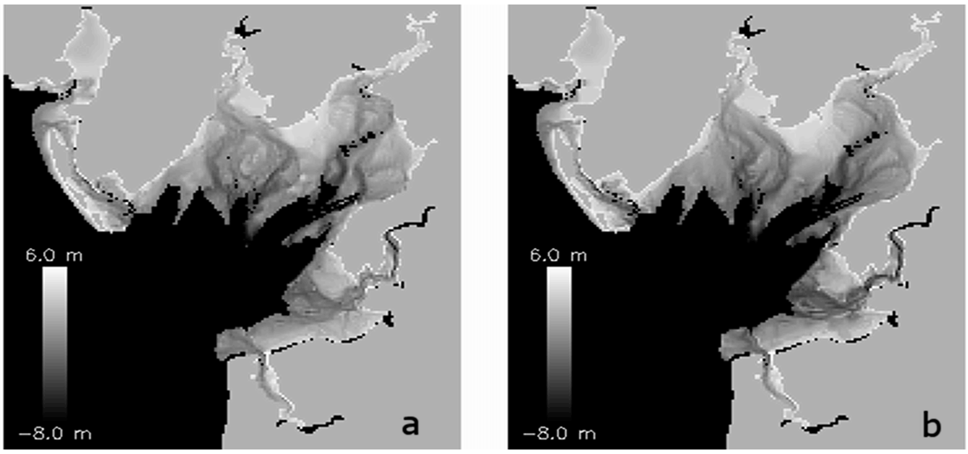

Figure 4: The bathymetry predicted for Morecambe Bay for November 2005 (a) including data assimilation (b) no data assimilation.

between the observation times; the swath bathymetry is for February 2005, and the LiDAR data were acquired in November 2005. There are some changes in the bathymetry during this period, which may affect the choice of parameter values. Another possibility is that the LiDAR data includes the tidal flats and shallower channels, and differences in the conditions here might suggest a lower sediment transport rate is most appropriate. Where available data are sparse it is possible that parameters selected by the calibration technique may be biased due to lack of fully representative data.

5.5 Improvements to bathymetric prediction using data assimilation

To demonstrate the utility of the data assimilation scheme, we compared the bathymetry predicted by using the model alone against that produced when observations were assimilated. A comparison of model runs for parameter values $A = 0.0012\,\mathrm{ms}^{-1}$ and $n = 3.0$ with and without data assimilation showed that the Brier Skill score improved from a value of 0.025 using the model alone to 0.35 when observations were assimilated. Assimilation of the available data thus provides a substantial improvement. Figure 4 shows a comparison of the final bathymetry for November 2005 (a) with, and (b) without data assimilation. It is clear that the channel positions are more similar to those of the LiDAR data when the observations are used to improve the model (see figure 3(b) for comparison). The changes in areas of deposition are also better represented with the assimilation, with old channel beds being filled in. There are still differences

A	n					
	2.2	**2.6**	**3.0**	**3.4**	**3.8**	**4.2**
0.0006		0.29	0.31	0.32	0.33	0.33
0.0012	0.33	0.34	0.35	0.34	0.32	0.30
0.0018	0.34	0.33	0.31	0.30	0.27	
0.0020	0.33	0.32	0.30	0.29		

Table 2: Brier skill scores obtained from LiDAR data for different combinations of parameters A and n.

between the model predictions and the LiDAR data, even when the assimilation is included. This is in part due to the partial area coverage of the waterline observations. The improvements are most marked in regions where we have observations. The assimilation has less effect on areas further away. As the data are also scattered in time over the 2 years of the model run (intervals between available waterlines vary from a couple of days to several months) not all of the changes in the bathymetry will be captured. However, even with the spatially and temporally sparse data, the improvement from the use of the data assimilation scheme is dramatic.

6 Joint state-parameter estimation

In section 5.4 we saw how the manual calibration of the Morecambe Bay model produced different parameter values depending on the type, coverage and survey time of the validation dataset. In this section we consider an alternative approach to morphodynamic parameter estimation using sequential data assimilation. Although data assimilation is predominantly used for state estimation, it is also possible to use the technique to estimate uncertain model parameters concurrently with the model state. We do this using the method of state augmentation. This approach has the advantage that it enables the parameters to be estimated and updated 'online' as new data become available rather than calibrating against historical data. It also has the potential of being able to estimate parameters that are expected to vary over time. Here, we describe the development of a novel hybrid sequential 3D-Var data assimilation algorithm for joint state-parameter estimation and demonstrate its application using an idealised 1D sediment transport model.

6.1 State augmentation

State augmentation is a conceptually simple technique that can in theory be applied to any standard data assimilation scheme; the parameters we wish to estimate are appended to the model state vector, the model state forecast equations are combined with the equations describing the evolution of the parameters and the chosen assimilation algorithm is simply applied to this new augmented system in the usual way (Jazwinski, 1970). This framework enables us to estimate the model parameters and update the predicted model state simultaneously, rather than treating as two individual processes; thereby saving on calibration time, facilitating better use of the available data and potentially delivering more accurate model forecasts. The technique has previously been successfully used in the treatment of systematic model error or forecast bias in atmosphere and ocean modelling using sequential and four-dimensional variational (4D-Var) assimilation methods (e.g. Bell et al., 2004; Dee, 2005; Griffith and Nichols, 2000; Martin et al., 1999). The review article by Navon (1997) discusses its application to parameter estimation in the context of meteorology and oceanography using 4D-Var, and more recently the approach has been employed for parameter estimation in simplified numerical models using variants of the extended and ensemble Kalman filters (Hansen and Penland, 2007; Trudinger et al., 2008).

In this study, we assume that the required parameters are constants, that is, they are not altered by the forecast model from one time step to the next. The parameter estimates will only change when they are updated by the data assimilation at each new analysis time. Note that this is not a necessary assumption, but one which is appropriate to the model we consider here. In this case, the evolution model for the parameters is given by the simple equation

$$\mathbf{p}_{k+1} = \mathbf{p}_k. \tag{7}$$

The parameter model (7) together with the model forecast equation (1) constitute an augmented state system model

$$\mathbf{w}_{k+1} = \tilde{\mathbf{f}}(\mathbf{w}_k), \qquad k = 0, 1, \ldots \tag{8}$$

where

$$\mathbf{w}_k = \begin{pmatrix} \mathbf{z}_k \\ \mathbf{p}_k \end{pmatrix} \in \mathbb{R}^{m+q}, \tag{9}$$

is the augmented state vector and

$$\tilde{\mathbf{f}}(\mathbf{w}_k) = \begin{pmatrix} \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k) \\ \mathbf{p}_k \end{pmatrix}, \tag{10}$$

with $\tilde{\mathbf{f}} : \mathbb{R}^{m+q} \longrightarrow \mathbb{R}^{m+q}$.

The equation for the observations (2) can be re-written in terms of the augmented system as

$$\mathbf{y}_k = \tilde{\mathbf{h}}(\mathbf{w}_k) + \boldsymbol{\delta}_k, \tag{11}$$

where $\tilde{\mathbf{h}} : \mathbb{R}^{m+q} \longrightarrow \mathbb{R}^{r_k}$, and

$$\tilde{\mathbf{h}}(\mathbf{w}_k) = \tilde{\mathbf{h}} \begin{pmatrix} \mathbf{z}_k \\ \mathbf{p}_k \end{pmatrix} = \mathbf{h}(\mathbf{z}_k), \quad k = 0, 1, \ldots \tag{12}$$

The cost function for the augmented system takes the same form as (3) but with the model state vector $\mathbf{z}_k$ replaced with the augmented state vector $\mathbf{w}_k$. Note that the augmented background vector $\mathbf{w}_k^b$ must now include *a priori* estimates of the both the model state variables and the parameters. Similarly, the augmented analysis $\mathbf{w}_k^a$ will now also include updated estimates of the model parameters $\mathbf{p}_k^a$ in addition to the updated state estimate, $\mathbf{z}_k^a$.

6.2 Error covariances

In basic state estimation the background error covariances govern how information is spread throughout the model domain, passing information from observed to unobserved regions and smoothing data if there is a mismatch between the resolution of the model and the density of the observations. Since it is not possible to observe the parameters themselves, the parameter estimates will depend upon the observations of the state variables. Successful application of the state augmentation technique relies strongly on the relationships between the parameters and state variables being well defined and assumes that we have sufficient knowledge to reliably prescribe them.

For the augmented system, the background error covariance matrix is redefined as

$$\mathbf{B}_k = \begin{pmatrix} \mathbf{B}_{\mathbf{zz}_k} & \mathbf{B}_{\mathbf{zp}_k} \\ (\mathbf{B}_{\mathbf{zp}_k})^T & \mathbf{B}_{\mathbf{pp}_k} \end{pmatrix}. \tag{13}$$

Here $\mathbf{B}_{\mathbf{zz}_k} \in \mathbb{R}^{m \times m}$ is the covariance matrix for the errors in the background state vector $\mathbf{z}_k^b$ at time t_k, $\mathbf{B}_{\mathbf{pp}_k} \in \mathbb{R}^{q \times q}$ is the covariance matrix for the errors in the parameter vector $\mathbf{p}_k^b$ and $\mathbf{B}_{\mathbf{zp}_k} \in \mathbb{R}^{m \times q}$ is the covariance matrix for the cross correlations between the background errors in the state and parameters.

For joint state-parameter estimation it is the state-parameter cross covariances, $\mathbf{B}_{\mathbf{zp}_k}$, that determine the influence of the observed data on the estimates of the unobserved parameters and therefore play a crucial role in the parameter updating. A good *a priori* specification of these covariances is fundamental to reliable joint state and parameter estimation. Since, by the nature of the problem, the statistics of these errors are not known exactly, they have to be approximated in some manner.

In initial experiments, we applied the same principle as for the state background error covariance matrix, and prescribed the state-parameter cross covariances a static functional form but this failed to produce reliable parameter estimates (Smith, 2010). Further investigation showed that we could yield accurate estimates of the both the parameters and the state variables by using a flow dependent structure for $\mathbf{B}_{\mathbf{zp}_k}$ (Smith et al., 2009a). Updating the full background error covariance matrix at every time step

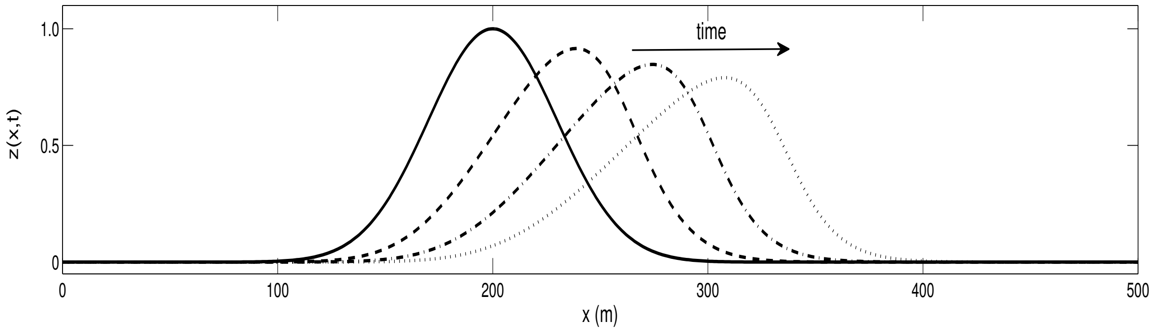

Figure 5: Evolution of the Gaussian bed-form over a 72 hour period for parameter values $A = 0.0018\,\mathrm{ms}^{-1}$ and $n = 3.4$.

can be computationally expensive and impractical for large scale systems. Importantly, the results of these experiments also showed that it is not necessary to explicitly propagate the full augmented matrix (13). We were able to recover accurate estimates of both the parameters and state variables by combining a time varying approximation of the state-parameter cross covariance matrix $\mathbf{B}_{\mathbf{zp}_k}$ with an empirical, static representation of the state background error covariance $\mathbf{B}_{\mathbf{zz}}$ and static parameter error covariance matrix $\mathbf{B}_{\mathbf{pp}}$.

This result led to the development of a novel hybrid assimilation algorithm. The new scheme combines ideas from 3D-Var and the extended Kalman filter (EKF) to construct a hybrid background error covariance matrix that captures the flow dependent nature of the state-parameter errors without the computational complexities of explicitly propagating the full system covariance matrix. The method is relatively easy to implement and computationally inexpensive to run. A 3D-Var approach is adopted for the state and parameter background error covariances; the matrices $\mathbf{B}_{\mathbf{zz}}$ and $\mathbf{B}_{\mathbf{pp}}$ are prescribed at the start of the assimilation and held fixed throughout as if the forecast errors were statistically stationary. The state-parameter cross-covariances are estimated using a simplified version of the EKF forecast step. This involves computing an approximation of the Jacobian of the state forecast model with respect to the parameters at each analysis time but since the number of parameters to be estimated is typically quite small this does not add significantly to the overall cost of the assimilation scheme. For the model presented in section 6.3 the Jacobian is computed using a local finite difference approximation. A detailed description of the development of this unique system is given in Smith (2010).

Key advantages of this new hybrid scheme are that the background error covariance matrix only needs to be updated at each new analysis time rather than at every model time step and it does not require the previous cross covariance matrices to be stored. It also avoids many of the issues associated with the implementation of fully flow dependent algorithms such as the extended and ensemble Kalman filters which include filter divergence, dynamic imbalances and rank deficiency (see e.g. Ehrendorfer (2007), Houtekamer and Mitchell (1998, 2005)).

6.3 The model

The potential applicability of the hybrid technique to morphodynamic modelling was investigated using an idealised 1D version of the sediment transport model employed in the Morecambe Bay model. The purpose of this study was to gain insight into some of the issues associated with practical implementation of the state augmentation framework in order to help guide the development of a data assimilation based state-parameter estimation system for the full model. We chose to use a relatively simple, low order system as a first step as this allowed us to concentrate on developing, testing and evaluating ideas rather than dealing with modelling complexities.

We use a generic test case consisting of a smooth, initially symmetric, isolated bed-form in an open channel. The bed level changes are governed by the 1D sediment conservation equation (cf. equation 5)

$$\frac{\partial z}{\partial t} = -\left(\frac{1}{1-\epsilon}\right)\frac{\partial q}{\partial x}\,, \tag{14}$$

where $z(x,t)$ is the bed height, t is time, q is the total (suspended and bedload) sediment transport rate in the x direction, and ϵ is the sediment porosity.

To calculate q we use the same simple power law equation as in the Morecambe Bay model (4) except that now the depth-averaged current $u = u(x,t)$ is one dimensional in the x direction and for ease of computation we assume that u takes only positive values. The sediment transport flux can then be written as

$$q = Au^n, \qquad \text{with } u \geq 0. \tag{15}$$

One of the primary reasons for choosing this simplified scenario is that, under certain assumptions, it is possible to derive an approximate analytical solution to (14) and this is useful for model validation purposes.

We assume that the amplitude of the bed-form is sufficiently small relative to the water depth such that any variation in the elevation of the water surface can be ignored. The water height, h, can then be taken to be constant. If we further assume that the water flux, F, is constant across the whole domain we can set $u(h - z) = F$. This enables us to express the sediment transport rate q as a function of bed height z rather than u. Equation (14) can then be re-written in the quasi-linear advection form

$$\frac{\partial z}{\partial t} + c(z)\frac{\partial z}{\partial x} = 0, \tag{16}$$

where the advection velocity of the bed $c(z)$ is now a function of the bed height z only. We note that the solution derived under these assumptions is only strictly valid when the migration speed of the bedform is slow relative to the flow velocity (Hudson, 2001).

To maintain numerical stability and ensure that the solution for z remains smooth and physically realistic we add a small diffusive term to the right hand side of (16). The model for the evolution of the bed-form is hence described by a nonlinear advection-diffusion equation. This is discretised using a hybrid semi-Lagrangian Crank-Nicolson algorithm as described in Smith (2010).

6.4 Experiments and results

The scheme was tested via a series of twin experiments using pseudo-observations with a range of spatial and temporal frequencies. For the purpose of these experiments, we assume that the values of h, F and ϵ are known and constant, but that the true values of the parameters A and n are uncertain, as in the full Morecambe Bay model. The water height and flux are specified as $h = 10.0\,\text{m}$, $F = 7.0\,\text{m}$ and the sediment porosity $\epsilon = 0.4$.

We generate a reference or 'true' solution by running the model on the domain $x \in [0, 500]$ with grid spacing $\Delta x = 1.0\,\text{m}$, time step $\Delta t = 30$ minutes, and parameter values $A = 0.0018\,\text{ms}^{-1}$ and $n = 3.4$. These are the parameter values that gave the best Brier skill score in the initial calibration of the Morecambe Bay model against the swath data. The initial profile for the true bathymetry is specified using a Gaussian function. The evolution of the bed-form over a 72 hour period is illustrated in figure 5.

The *a priori* bathymetry for the assimilation is given by the same Gaussian function but with different scaling factors so that it is a different height, width and in a slightly different starting position to the truth. The state background error covariance matrix $\mathbf{B_{zz}} = b_{ij}$ is modelled using the isotropic correlation function (Rodgers, 2000)

$$b_{ij} = \sigma_b^2 \rho^{|i-j|}, \quad i, j = 1, ..., m, \tag{17}$$

with $\rho = \exp(-\Delta\text{x}/\ell)$ where ℓ is a correlation length scale that is defined empirically. For the experiments described here, ℓ is set at four times the observation spacing.

The parameter vector is given by

$$\mathbf{p}_k = \begin{pmatrix} A_k \\ n_k \end{pmatrix}. \tag{18}$$

The parameter error covariance matrix $\mathbf{B_{pp}}$ is the (2×2) matrix

$$\mathbf{B_{pp}} = \begin{pmatrix} \sigma_A^2 & \sigma_{An} \\ \sigma_{nA} & \sigma_n^2 \end{pmatrix}. \tag{19}$$

The diagonal elements σ_A^2 and σ_n^2 are the error variances for A and n respectively and $\sigma_{An} = \sigma_{nA}$ and is the covariance between the errors in A and n. Here, σ_A^2 and σ_n^2 are set equal to the square of the error in the chosen initial estimates and σ_{An} is specified according to the direction of these errors as described in Smith et al. (2009b); Smith (2010).

The state-parameter cross covariance matrix $\mathbf{B}_{\mathbf{zp}_k}$ is recalculated at each new analysis time by computing the Jacobian of the state forecast model with respect to the parameters using a local finite difference approximation as described in Smith (2010) and Smith et al. (2011).

Observations of the bed height $z(x, t)$ are generated by sampling the true solution on a regularly spaced grid and are assimilated sequentially at regular time intervals. The space and time frequency of the observations remains fixed for the duration of each individual assimilation experiment but is varied between experiments as described in the discussion of our results below. The augmented cost function is minimised iteratively using a quasi-Newton descent algorithm (Gill et al., 1981). At the end of each assimilation cycle the current values of A and n in the model are replaced with the new estimates, $\mathbf{p}^a$, the current model state is updated with the state analysis, $\mathbf{z}^a$, and the model is forecast forward to the next observation time.

6.4.1 Perfect observations

At first, we assume that the observations are perfect. The initial parameter estimates for A and n are taken as $A = 0.006\,\mathrm{ms}^{-1}$ and $n = 4.2$ which are the lower and upper extremes of the calibration range used in the Morecambe Bay model. The observation and background error variances are set at $\sigma_o^2 = 0.01$ and $\sigma_b^2 = 0.1$ respectively and the parameter cross correlation ρ_{An} is taken to be strongly negative.

Figures 6 and 7 show the parameter updates when observations are taken at varying temporal and spatial frequencies. Figure 6 shows the effect of reducing the temporal frequency of the observations from every 2 to every 24 hours. The spatial frequency of the observations is fixed at 25.0 m. The speed of convergence of the estimates decreases as the time frequency of the observations decreases but the scheme successfully retrieves the true values of A and n to a high level of accuracy in all cases. Even when the observation frequency is further reduced to every 48 hours (2 days) the estimates eventually converge to close to their exact values after around 6 days (not shown).

Decreasing the spatial frequency has a similar effect. Figure 7 shows the parameter updates produced when the grid spacing between the observations is increased from every $10\Delta x$ to every $50\Delta x$. Once again, the true parameter values are successfully estimated to high accuracy. There is a large difference in the rate of convergence when the observation interval is doubled from $25\Delta x$ to $50\Delta x$. If this increased further to $75\Delta x$ the scheme fails to recover the correct parameter values. In this case the spatial resolution of the observations is too low to be able to consistently capture key features of the bed-form, in particular the height and position of its peak. This information is crucial to being able to accurately track the movement of the bed across the domain and in turn to reliably estimate A and n.

6.4.2 Imperfect observations

We examined the impact of observation errors on the scheme by adding random Gaussian noise to the observations. In these examples, we reverse the direction of the parameter errors; the initial estimate for n is taken as $n = 2.2$ which is the lower limit of the range used in the Morecambe Bay calibration; parameter A is set at $A = 0.018\,\mathrm{ms}^{-1}$ which is ten times greater than the true value. Observations were taken at 25 m intervals and assimilated every 2 hours. The background and observations are now given equal weight with $\sigma_b^2 = \sigma_o^2$. The parameter error variances and cross covariance are defined as described above for the perfect observation case.

Figure 8 shows the parameter estimates produced when noise with error variances $\sigma_o^2 = 0.01$ and $\sigma_o^2 = 0.1$ was added to the observations. An error variance of $\sigma_o^2 = 0.1$ corresponds to 10% of the maximum bed height and is considered to be a realistic representation of measurement error. Even though the parameter estimates are now noisy they are fluctuating about the true values and lie within the bounds of the uncertainty placed on the observations. The amplitude of the oscillations is similar in both cases but are slightly more erratic when $\sigma_o^2 = 0.1$. This is likely to be due to the model state estimate used in the Jacobian approximation for $\mathbf{B}_{\mathbf{zp}_k}$ being more noisy.

We found that smoother, more accurate parameter estimates could be obtained by averaging over a moving time window as the assimilation is running, as illustrated in figure 9. Averaging is started at $t = 24$ hours to allow time for the scheme to settle. Here, we use a time window of 6 hours. The smoothed estimates are extremely close to those obtained in the perfect observation case.

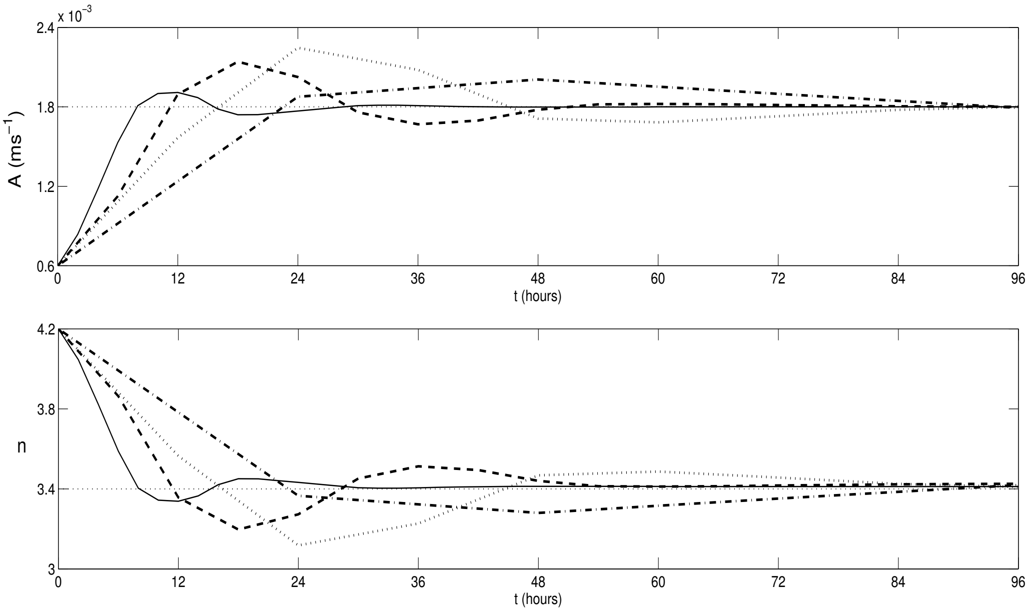

Figure 6: Varying the time frequency of observations. Spatial frequency of observations is fixed at 25.0 m. Parameter updates for initial estimates $A_0 = 0.0006\text{ms}^{-1}$ (top) and $n_0 = 4.2$ (bottom). *Solid line* - observations every 2 hours; *dashed line* - observations every 6 hours; *dotted line* - observations every 12 hours; *dot-dash line* - observations every 24 hours. The true parameter values $A = 0.0018\,\text{ms}^{-1}$ and $n = 3.4$ are given by the horizontal dotted line.

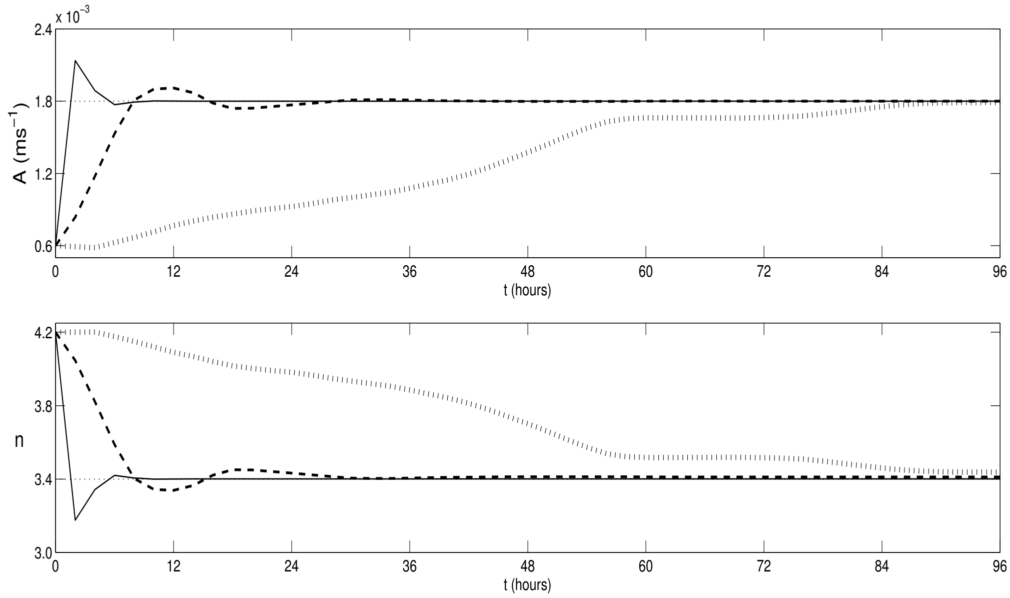

Figure 7: Varying the spatial frequency of observations. Temporal frequency of observations is fixed at 2 hour intervals. Parameter updates for initial estimates $A_0 = 0.0006\text{ms}^{-1}$ (top) and $n_0 = 4.2$ (bottom). *Solid line* - observations every $10\Delta x$; *dashed line* - observations every $25\Delta x$; *dotted line* - observations every $50\Delta x$. The true parameter values $A = 0.0018\,\text{ms}^{-1}$ and $n = 3.4$ are given by the horizontal dotted line.

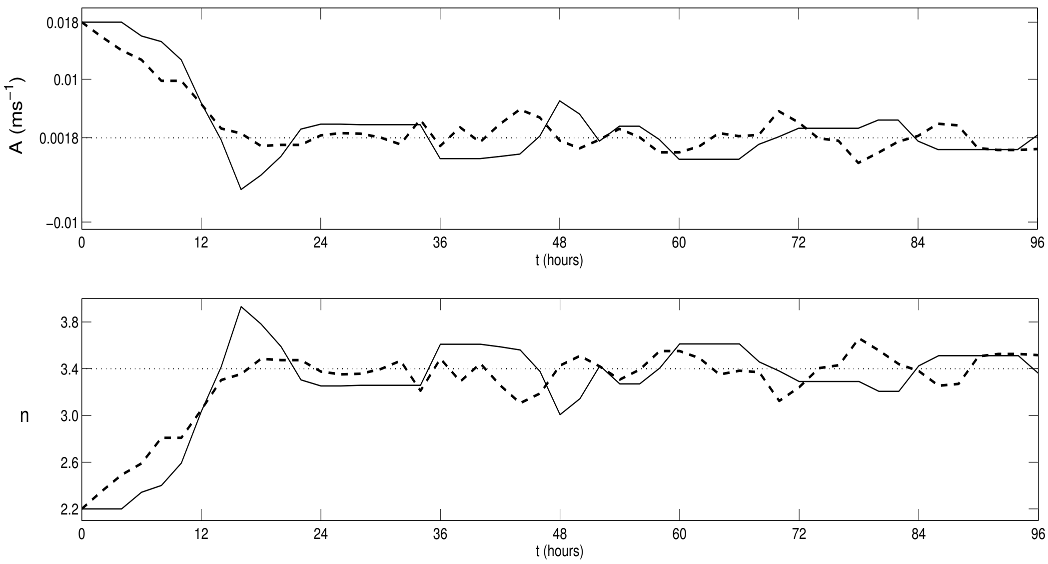

Figure 8: Parameter updates for initial estimates $A_0 = 0.018\text{ms}^{-1}$ (top) and $n_0 = 2.2$ (bottom) with noisy observations assimilated every 2 hours at 25 m intervals. *Solid line*, $\sigma_o^2 = 0.01$; *dashed line*, $\sigma_o^2 = 0.1$. The true parameter values $A = 0.0018\,\text{ms}^{-1}$ and $n = 3.4$ are given by the horizontal dotted line.

We have tested the efficacy of our hybrid method in a variety of scenarios in addition to those presented here. As we would expect, there are bounds on the success of the approach. As is the case with any data assimilation scheme, the quality of the analysis is highly dependent on the quality of the information fed into the assimilation algorithm; if observational data are too infrequent or too noisy or if the initial state and parameter background estimates are particularly poor then we cannot expect the scheme to yield reliable results. Generally, we saw a deterioration in the parameter estimates and model state predictions as the quality and frequency of the observation decreased. The estimated error variances and the relative weighting between the background and observations were also found to be extremely important to the success of the method. Further experiments with more detailed results and discussions are given in Smith et al. (2009b); Smith (2010) and Smith et al. (2011).

7 Conclusions

Up-to-date knowledge of near-shore coastal bathymetry is important in flood prediction and risk management. In this paper we have demonstrated how the application of data assimilation techniques in morphodynamic modelling can be used to generate forecasts of bathymetry that are more accurate than using a model alone by (1) producing improved estimates of the current model bathymetry, and (2) providing estimates of uncertain morphodynamic model parameters.

Section 5 described the implementation of a sequential 3D-Var data assimilation scheme for model state estimation in Morecambe Bay. The results of this section illustrated how data assimilation can be used to greatly improve model predictions of the evolution of the bathymetry in the bay. Even though the model used here is simple relative to state of the art engineering models, the inclusion of partial observations of bathymetry taken at irregular and sometimes large intervals is sufficient to produce a reasonable match to the true state of the bathymetry after a 2 year model run. This suggests that extremely complex models are not necessarily required, and that more computationally efficient, simplified models are adequate if a data assimilation scheme is employed. A key difficulty with morphodynamic models is their dependence on parameters whose values are typically uncertain and often dependent on the characteristics of the study site. In the Morecambe Bay study, the model parameters were set by running the model with different parameter combinations and examining the results to find the values that produced the closest match to the observed data. This is a standard approach but is time consuming and can produce biased estimates if

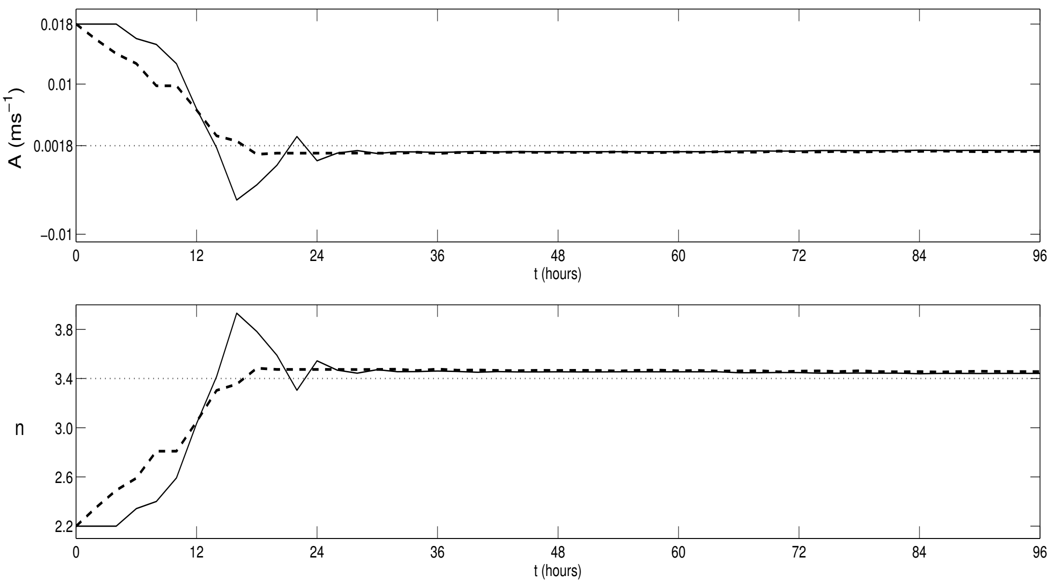

Figure 9: Time averaged parameter updates for initial estimates $A_0 = 0.018\text{ms}^{-1}$ (top) and $n_0 = 2.2$ (bottom) with noisy observations assimilated every 2 hours at 25 m intervals. *Solid line*, $\sigma_o^2 = 0.01$; *dashed line*, $\sigma_o^2 = 0.1$. Averaging is started at $t = 24$ hours.The true parameter values $A = 0.0018\,\text{ms}^{-1}$ and $n = 3.4$ are given by the horizontal dotted line.

the observational data available for calibration is limited. The parameters providing the best results using our calibration dataset did not produce the best results when the modelled bathymetry was compared with validation data for the whole of the bay. This discrepancy is possibly due to the calibration dataset covering only the deeper parts of the bay and channels, thus biasing the parameter selection towards values that more closely reproduced the conditions in these areas.

In section 6 we explained how the problem of morphodynamic model parameter estimation can be addressed using sequential data assimilation. We introduced the technique of state augmentation and described how the approach can be used to estimate poorly known model parameters jointly with the model state variables. The augmented assimilation framework enables us to estimate the model state and parameters simultaneously, rather than treating state and parameter estimation as two separate problems. The approach offers an attractive alternative to traditional calibration techniques, allowing more efficient use of observational data and potentially producing improved model state forecasts. A particular advantage of sequential data assimilation is that observational data can be used as they arrive in real time. This allows for the inclusion of information from new observations that become available after earlier observations have been assimilated. We gave details of the development of a novel hybrid 3D-Var assimilation algorithm and demonstrated the efficacy of the new method using an idealised 1D version of the sediment transport model implemented for Morecambe Bay. The results were excellent; the scheme is able to successfully recover the true parameter values, even when the observational data are noisy, and this has a positive impact on the forecast model. In the experiments with noisy observations we found averaging the parameters over a moving time window to be very effective at improving the accuracy of the estimates. The results of this study are extremely promising and suggest that there is great potential for the use of data assimilation based joint state and parameter estimation in coastal morphodynamic modelling.

The hybrid algorithm developed in this work offers an effective and versatile approach to approximating the state-parameter cross covariances demanded by the augmented system. The scheme is relatively simple to implement and computationally inexpensive to run. In this paper we have focused on application of the approach in the context of morphodynamic modeling, but the method has also been applied in a range of simple dynamical systems models with equally positive results (Smith et al., 2010; Smith, 2010). We expect that this new methodology will be easily transferable to larger more realistic models with more complex parameterisations.

The next step is to implement a joint state-parameter estimation data assimilation scheme in the Morecambe Bay model. This will offer number of challenges but will provide a useful real-world test case for the hybrid technique. The opportunity to further explore some of the practical issues identified in this initial study will lead to an improved understanding of the process of taking our new method from theory into operational practice.

The most complex part of applying the method to the 2D model will be construction of the augmented background error covariance matrix. The success of the state augmentation approach relies heavily on the relationship between the state and parameters (as described by the state-parameter cross covariances) being well defined. The approximation of the state-parameter covariance matrix $\mathbf{B}_{\mathbf{z}\mathbf{p}_k}$ used in the hybrid scheme requires computation of the Jacobian of the forecast model with respect to the model parameters. For the 1D model, this was calculated using a local finite difference approximation. Because the number of parameters was small and the dimension of the state vector relatively low, this calculation did not add a significantly to the overall cost of the assimilation scheme. This calculation will be computationally intensive for the full model but does not render the approach infeasible.

The definition of the parameter background error covariance matrix $\mathbf{B}_{\mathbf{pp}}$ is relatively simple but its specification requires some level of *a priori* knowledge of the parameter error statistics and an understanding of the relationship between individual parameters. Difficulties can arise when parameters exhibit strong interdependence, in this case the prescription of the parameter correlations can have a significant effect on the accuracy of the estimates obtained. In our simple model experiments we knew the magnitude and direction of the errors in our initial parameter estimates and were able to set the parameter error variances and cross covariances accordingly. In practical situations, where the true statistics of the errors are not known, a simple sensitivity analysis can be used to help identify the existence and degree of interdependence between parameters.

A further potential issue is the type and frequency of availability of observational data. Our 1D model experiments found that the quality of the state and parameter estimates varied with the spatial and temporal frequency of the observations. Here, we used synthetic observations that were direct, evenly spaced and assimilated at regular time intervals. The Morecambe Bay model is run on a much larger domain and over much longer timescales. The observational data are indirect and spatially and temporally sparse. Work with this model will provide a useful insight into whether the space and time frequencies of observations available on an operational scale are sufficient for a combined state-parameter estimation scheme to be effective. It could also potentially aid the design of an optimal observation network for coastal monitoring.

Ultimately, we hope to use the experience gained from this research to help guide the application of data assimilation based state and parameter estimation in an operational setting. The extension of the Morecambe Bay 3D-Var assimilation scheme to concurrent state-parameter estimation is an important intermediate step that will enable us to assess the feasibility of our hybrid assimilation algorithm in the context of operational scale coastal morphodynamic modelling, and to evaluate whether the approach offers a viable, cost effective alternative to the model calibration techniques currently in use.

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